

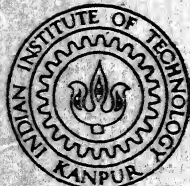
SYNTHESIS OF CAM-FOLLOWER SYSTEMS WITH ROLLING CONTACT

By

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DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
JANUARY, 1980

SYNTHESIS OF CAM-FOLLOWER SYSTEMS WITH ROLLING CONTACT

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By
RANJEET PRASAD YADAV

to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
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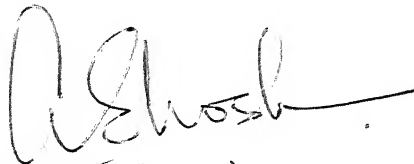
Dedicated to

"My Parents"

whose unfathomable love and affection have
always been a source of inspiration for me.

CERTIFICATE

This is to certify that the thesis entitled
"Synthesis of Cam-Follower Systems with Rolling Contact"
by R.P. Yadav is a record of work carried out under my
supervision and has not been submitted elsewhere for a
degree.



(Dr. A. Ghosh)
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ACKNOWLEDGEMENT

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R.P. Yadav

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NOMENCLATURE

ϕ	= Angle of rotation of cam
L	= Total lift of the follower
L_r	= Lift contributed by rolling action
L_s	= Lift contributed by rolling-cum sliding action
ϕ_r	= Angle of cam rotation for the rolling part
ϕ_s	= Angle of cam rotation for the sliding part
N	= Normal force between the cam and the follower surface
P_o	= Initial load on the follower
P	= External load on the follower (spring force)
R_1, R_2	= Horizontal reaction forces at the follower and the guide
μ	= Coefficient of friction in the guide
μ_i	= Coefficient of friction at the cam follower interface
h	= Overhang of the follower beyond the guide
l	= Bearing length of the guide
d	= Diameter of the follower shaft
α	= Instantaneous pressure angle
k	= Stiffness of compression spring
ρ_1	= Slope of the displacement diagram at the end of rolling contact
ρ_o	= Slope of the displacement diagram at the beginning of sliding contact

SYNOPSIS
of the
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In the present work, cam and the corresponding follower contours have been synthesised for pure rolling contact during rise period of a dwell-rise-dwell-return cam. It is found that the total lift of the follower has to be achieved partially by rolling contact and rest by rolling cum sliding action. Therefore, for sliding portion also cam contour is synthesised, assuming the follower surface to be flat. Finally for the rise period the forces on the cam follower mechanism are analysed to estimate the work losses in the guide and at the cam follower interface during sliding contact. The computed work-losses give the idea about optimal sharing for total lift by rolling contact and sliding action.

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Present - day requirements demand that machine components move in a prescribed exact path. Connected members alone can rarely fulfill these requirements. It is, therefore, necessary to resort to the use of miscellaneous contour surfaces called cams. Moreover cam-mechanism has the virtue of being simple to design for prescribed input-output characteristics. For a given motion the synthesis of other types of mechanism is generally more complicated.

The use of cams makes it possible to obtain an unlimited variety of motion and when certain basic requirements are followed, cams perform satisfactorily for a sufficiently long period of time.

Due to above mentioned characteristics of cam-follower mechanism, it finds its use very frequently in all types of machines.

Various types of follower are in use. Important amongst them are knife edge follower, roller follower, flat face follower and curved face follower. Its disposition may be either radial or offset with respect to the centre of rotation of the cam and its motion is either translatory or oscillatory.

In most of the cases cams used are plate or disk type.

1.2 DESIGN CONSIDERATIONS FOR THE SYNTHESIS OF CAM - FOLLOWER SYSTEM

While designing a cam-follower system, the detailed study of the contours of cam and follower and the characteristics of velocity and acceleration curves must be made. The undermentioned factors must be kept in mind for the design work:

The cam almost always runs at a constant speed which is established by the desired values of the follower displacement, velocity and acceleration.

The cam-contour should be made smooth with no abrupt change in its curvature. The pressure angle should also be kept to a minimum value.

The cam size should be as small as possible to minimise the cam-follower sliding velocity, surface wear and torque. It also results in improved balancing of the cam.

The follower acceleration at high speeds should be as low as possible to keep the inertia forces and thus stresses low. The selection of the shape of the follower acceleration curve is also important because at high speeds the noise, surface wear and vibration of the cam-follower system is largely dependent on it.

The moving parts of the cam-follower mechanism should be made as light in weight and as rigid as possible.

Manufacturing methods and accuracy of cutting and inspection are of paramount importance in assuring the anticipated performance. A very small surface errors may cause high stresses and vibrations in the follower linkages at high speed.

1.3 PREVIOUS WORK

In the field of cam-follower design for pure rolling contact motion, very little work has been done. One of the papers available is by G.R. Veldkamp [1]. In this paper for flat face follower, the cam contour for pure rolling contact motion is developed analytically. However the work is far from being complete as the applicability of the system proposed by him is limited to the cases with oscillatory rotational motion of the cam. Moreover the type of the follower motion which is one of the basic criteria in cam-design becomes dependent.

1.4 OBJECTIVE AND SCOPE OF THE PRESENT WORK

The present work attempts a solution for determining cam-contour and the corresponding follower surface for pure rolling contact motion, given the total lift and the corresponding angle of rotation of the cam.

Instead of assuming the follower to be flat the surface of the follower has also been synthesised along

with the cam contour from the point of view of optimal performance.

It was attempted with arbitrary follower motion during the rise period of the dwell-rise-dwell-return cam, but some constraints are felt for the transmission of motion from cam to the follower.

One of the constraints is that the follower motion during the rise period must have the increasing slope throughout. Under the present situation (dwell-rise-dwell-return cam), therefore, it is not possible to provide pure rolling motion for the entire rise period. Hence the rise period consists of initial rolling followed by a portion of sliding.

Other constraint comes from the fact that there must be some limitations to the pressure angle.

CHAPTER 2

CAM - FOLLOWER SYNTHESIS FOR PURE ROLLING CONTACT

In this chapter, the cam contour and the follower surface has been synthesised for pure rolling contact. The criteria used for rolling action is discussed in Section 2.1.

2.1 CRITERIA FOR PURE ROLLING

Two bodies 1 and 2 are assumed to move relative to each other, as shown in the Fig. 2.1. Body 1 is taken to be the driver and body 2 the driven.

P is the point of contact. The adjacent points on bodies 1 and 2 are designated as P_1 and P_2 respectively. The velocity of point P_2 is related to that of point P_1 through the following relation:

$$\vec{V}_{P_2} = \vec{V}_{P_1} + \vec{V}_{P_2/P_1} \quad (2.1)$$

\vec{V}_{P_2/P_1} is the velocity of point P_2 relative to point P_1 .

For pure rolling motion, the relative velocity term must be zero

$$\text{i.e., } \vec{V}_{P_2/P_1} = \vec{V}_{P_1/P_2} = 0 \quad (2.2)$$

Hence equation 2.1 becomes

$$\vec{V}_{P_2} = \vec{V}_{P_1} \quad (2.3)$$

This gives the criteria for pure rolling motion. It states that the points of contact, considered on either of the two bodies, must have the same velocity.

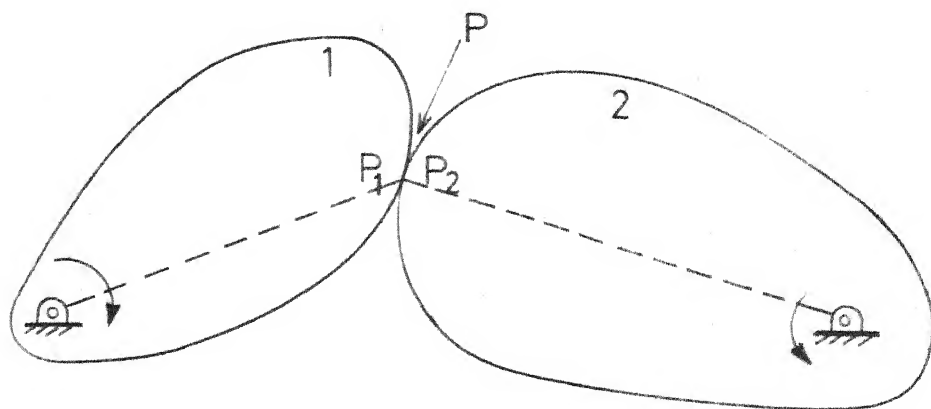


FIG. 2.1

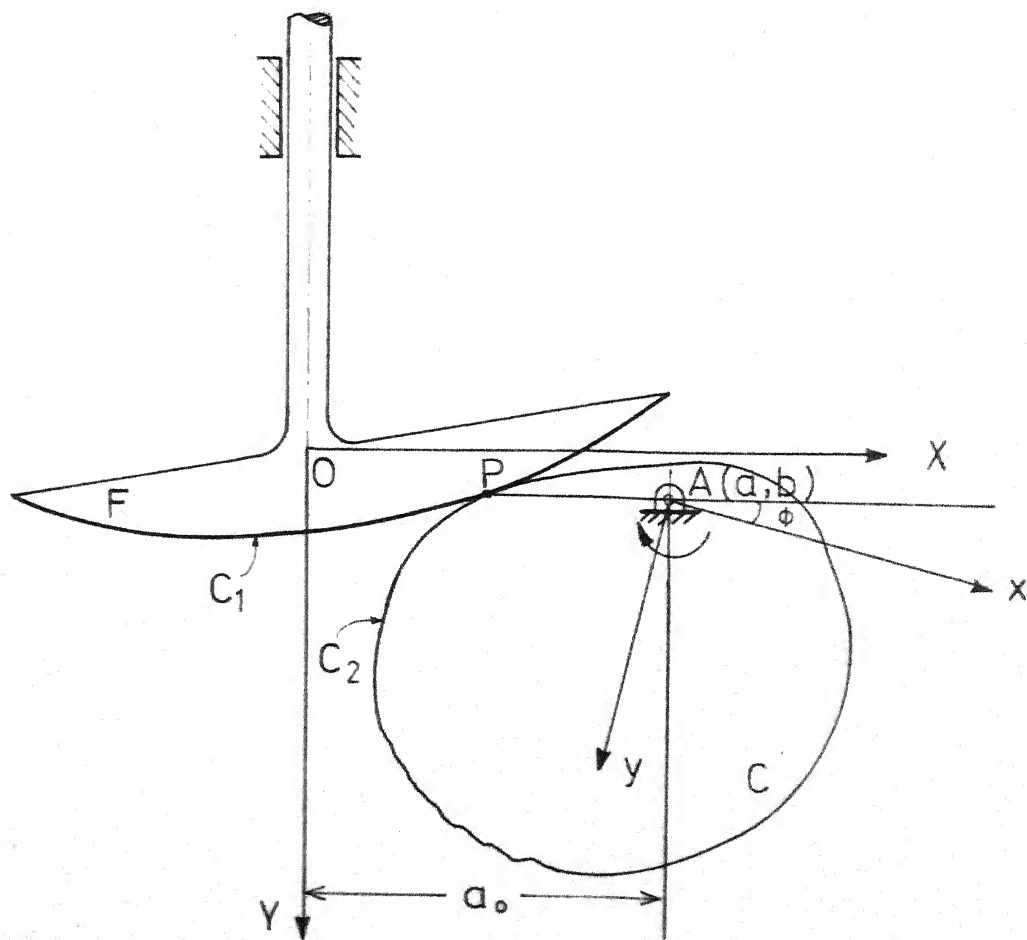


FIG. 2.2

Here, for pure rolling contact motion the cam contour and the follower surface will be synthesised, using the criteria for rolling as discussed in Section 2.1. The analysis is as follows:

The schematic arrangement of a cam-follower system is shown in the Fig. 2.2. ϕ is the common plane of symmetry of the cam 'C' and the follower 'F', and it passes through the axis of the shaft of the follower. ϕ_1 and ϕ_2 are two planes coinciding with ϕ and rigidly connected with 'F' and 'C' respectively.

Now, for the motion, "'C' rolls without sliding on the foot of 'F' ", it is assumed that the centre of this motion be represented by C_1 in plane ϕ_1 and by C_2 in plane ϕ_2 respectively. Thus C_1 is the intersection of the follower-surface with plane ϕ_1 and C_2 is the intersection of the cam-surface with plane ϕ_2 , which is nothing but the cam-contour.

Referring to Fig 2.2,

XOY is the co-ordinate system attached to the follower which is assumed to be fixed. xoy is the co-ordinate system attached to the cam and having its origin at the point A which is the centre of rotation of the cam. XOY and xoy axes have the same orientation. (a, b) are the co-ordinates of point A with respect to the fixed axis XOY.

\emptyset represents the angle of cam rotation, \emptyset being taken as positive when measured clockwise.

If the point of contact 'P' between the cam and the follower surfaces has the co-ordinates (x, y) in the xoy co-ordinate system and (X, Y) in the XOY system, then as per law of co-ordinate transformation

$$\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \emptyset & -\sin \emptyset & a \\ \sin \emptyset & \cos \emptyset & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad (2.4)$$

$$\begin{aligned} \text{or } X &= x \cos \emptyset - y \sin \emptyset + a \\ \text{and } Y &= x \sin \emptyset + y \cos \emptyset + b \end{aligned} \quad (2.5)$$

Cam-follower motion can be considered equivalent to the following motion:

"Follower is assumed to be fixed, cam rotates as well as the point A (centre of rotation of cam) translates along a straight-line which is parallel to the axis of the follower. The translatory motion of the point 'A' is similar to the required motion of the follower." Now, if the point P_1 on the follower is adjacent to P, then

$$\vec{V}_{P_1} = 0, \text{ because follower is fixed.}$$

This demands that

$$\dot{X} = 0 \quad (2.6)$$

$$\text{and } \dot{Y} = 0 \quad (2.7)$$

\dot{X} and \dot{Y} represent time derivatives of X and Y respectively.

Now, differentiating first equation of (2.5) with respect to time, it gives

$$\dot{X} = \dot{x} \cos \phi - x \sin \phi \frac{d\phi}{dt} - \dot{y} \sin \phi - y \cos \phi \frac{d\phi}{dt} + \frac{da}{dt} \quad (2.8)$$

But $\frac{d\phi}{dt} = \omega$, ω being the angular velocity of the cam

$$\text{and } \frac{da}{dt} = \frac{da}{d\phi} \frac{d\phi}{dt}$$

$$= \frac{da}{d\phi} \cdot \omega$$

$$= \omega \cdot a', \text{ where } a' = \frac{da}{d\phi}$$

Therefore, (2.8) becomes

$$\dot{X} = \dot{x} \cos \phi - x \cdot \omega \sin \phi - \dot{y} \sin \phi - y \omega \cos \phi + \omega a' \quad (2.9)$$

$$\text{But } \dot{x} = 0$$

$$\text{and } \dot{y} = 0$$

This is because of the fact that the velocity of any point on the cam relative to xoy axis which is attached to cam has to be zero.

Hence, equation 2.9 with the use of (2.6) yields

$$\omega(x \sin \phi + y \cos \phi - a') = 0 \quad (2.10)$$

But ω is not zero, therefore (2.10) gives

$$x \sin \phi + y \cos \phi = a' \quad (2.11)$$

Similarly, differentiating second equation of (2.5) with respect to time and using condition (2.7), it is obtained that

$$-x \cos \theta + y \sin \theta = b', \quad (2)$$

$$\text{where } b' = \frac{db}{d\theta}$$

Thus, from equations 2.11 and 2.12

$$x \sin \theta + y \cos \theta = a'$$

$$-x \cos \theta + y \sin \theta = b' \quad (2)$$

Again,

when the point 'P₂' on the cam is adjacent to P (Fig. 2.3), then

$$\vec{V}_{P_2} = \vec{V}_A + \vec{V}_{P_2/A} \quad (2.1)$$

\vec{V}_{P_2} and \vec{V}_A are respectively the velocity of points P₂ and A relative to the axis XOY and $\vec{V}_{P_2/A}$ is the velocity of point P₂ relative to the point A on the cam

But

$$\vec{V}_{P_2/A} = \frac{d\vec{\xi}}{dt}, \quad (\text{Fig. 2.3})$$

Where

$\vec{\xi} = x \vec{i} + y \vec{j}$, \vec{i} and \vec{j} are unit vectors along axes x and y respectively. It is to be noted that they are rotating vectors.

Hence,

$$\begin{aligned} \vec{V}_{P_2/A} &= \frac{d}{dt} (x \vec{i} + y \vec{j}) \\ &= (\dot{x} \vec{i} + y \dot{\vec{i}}) + (x \dot{\vec{i}} + y \dot{\vec{j}}) \end{aligned} \quad (2.15)$$

$$\text{But } \dot{\vec{i}} = \omega \vec{j}$$

$$\text{and } \dot{\vec{j}} = -\omega \vec{i}$$

Moreover $\dot{x} = 0 = \dot{y}$

Hence (2.15) becomes

$$\vec{V}_{P_2/A} = \vec{\omega} \times (x \vec{i} + y \vec{j}) \quad (2.16)$$

But $\vec{\omega}$ can be written as $\vec{\omega} = \omega \vec{k}$, \vec{k} being a unit vector along z - axis and xyz forms a right handed co-ordinate system in space.

Thus, equation 2.16 yields

$$\begin{aligned} \vec{V}_{P_2/A} &= \omega \vec{k} \times (x \vec{i} + y \vec{j}) \\ \text{or } \vec{V}_{P_2/A} &= \omega (x \vec{j} - y \vec{i}) \end{aligned} \quad (2.17)$$

Now,

$$\vec{V}_A = \dot{a} \vec{u} + \dot{b} \vec{v} \quad (2.18)$$

\vec{u} and \vec{v} are unit vectors along co-ordinate axes OX and OY respectively.

Also, for pure rolling between the cam and the follower surfaces

$$\vec{V}_{P_2} = \vec{V}_{P_1}$$

$$\text{As } \vec{V}_{P_1} = 0$$

$$\text{Hence } \vec{V}_{P_2} = 0 \quad (2.19)$$

substituting (2.16), (2.18) and (2.19) in (2.14), it gives

$$(\dot{a} \vec{u} + \dot{b} \vec{v}) + \omega (x \vec{j} - y \vec{i}) = 0 \quad (2.20)$$

But law of transformation gives

$$\begin{bmatrix} \vec{i} \\ \vec{j} \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \vec{u} \\ \vec{v} \end{bmatrix}$$

$$\begin{aligned} \text{i.e., } \vec{i} &= \vec{u} \cos \phi + \vec{v} \sin \phi \\ \text{and } \vec{j} &= -\vec{u} \sin \phi + \vec{v} \cos \phi \end{aligned} \quad (2.21)$$

Substituting for \vec{i} and \vec{j} from (2.21) in (2.20), it becomes

$$\begin{aligned} (\dot{a} \vec{u} + \dot{b} \vec{v}) + \omega \cdot x \cdot (-\vec{u} \sin \phi + \vec{v} \cos \phi) \\ - \omega \cdot y \cdot (\vec{u} \cos \phi + \vec{v} \sin \phi) = 0 \end{aligned}$$

$$\text{or } (\dot{a} - x \omega \sin \phi - y \omega \cos \phi) \vec{u} + (\dot{b} + x \omega \cos \phi - y \omega \sin \phi) \vec{v} = 0$$

which gives

$$\begin{aligned} \dot{a} - \omega (x \sin \phi + y \cos \phi) &= 0 \\ \text{and } \dot{b} + \omega (x \cos \phi - y \sin \phi) &= 0 \end{aligned} \quad (2.22)$$

$$\begin{aligned} \text{But } \dot{a} &= \omega \cdot a' \\ \text{and } \dot{b} &= \omega \cdot b' \end{aligned}$$

Hence equations 2.22 simplify to the following form:

$$\begin{aligned} x \sin \phi + y \cos \phi &= a' \\ -x \cos \phi + y \sin \phi &= b' \end{aligned} \quad (2.23)$$

It is to be noted that equations 2.13 and 2.23 are the same.

Thus,

$$x \sin \emptyset + y \cos \emptyset = a' \quad (2.24)$$

$$\text{and } -x \cos \emptyset + y \sin \emptyset = b' \quad (2.25)$$

Now, Equations 2.25 and 2.5 give

$$a - X = b'$$

$$\text{or } X = a - b' \quad (2.26)$$

Similarly, Equations 2.24 and 2.5 give

$$Y - b = a'$$

$$\text{or } Y = a' + b \quad (2.27)$$

Thus,

$$X = a - b' \quad (2.28)$$

$$\text{and } Y = a' + b$$

The equations 2.28 are the parametric equations of the follower surface ' C_1 '.

Now, multiplying (2.24) by $\sin \emptyset$ and (2.25) by $\cos \emptyset$, and then subtracting the later from former, it gives

$$x = a' \sin \emptyset - b' \cos \emptyset \quad (2.29)$$

Similarly, multiplying (2.24) by $\cos \emptyset$ and (2.25) by $\sin \emptyset$ and then adding, it gives

$$y = a' \cos \emptyset + b' \sin \emptyset \quad (2.30)$$

Thus,

$$x = a' \sin \emptyset - b' \cos \emptyset \quad (2.31)$$

$$y = a' \cos \emptyset + b' \sin \emptyset$$

This set of equations represents the parametric equation of the cam-surface ' C_2 '.

It is assumed that as the cam rotates, its centre of rotation 'A' translates along a straight line 'l'. Line 'l' is so chosen that it is parallel to the Y - axis which is assumed to be the axis of follower. Under this condition, 'a' becomes a constant and thus $a' = 0$.

Taking $a = a_0$, (2.28) becomes

$$\begin{aligned} X &= a_0 - b' \\ \text{and } Y &= b \end{aligned} \quad (2.32)$$

This set of equations represents the parametric equation of the follower surface.

Also, the equations 2.31 give

$$\begin{aligned} x &= -b' \cos \phi \\ y &= b' \sin \phi \end{aligned} \quad (2.33)$$

If polar co-ordinate is used for the cam-profile in the form, (Fig. 2.4)

$$\begin{aligned} x &= \xi \cos \theta \\ \text{and } y &= \xi \sin \theta \end{aligned} \quad (2.34)$$

(2.33) and (2.34) combine to give

$$-b' \cos \phi = \xi \cos \theta \quad (2.35)$$

$$\text{and } b' \sin \phi = \xi \sin \theta \quad (2.36)$$

Squaring equations 2.35 and 2.36, and then adding gives

$$\begin{aligned} (b')^2 &= \xi^2 \\ \text{or } \xi &= b' (\phi) \end{aligned} \quad (2.37)$$

This is the equation of the cam-contour in polar co-ordinate and this form is found to be convenient for drawing purpose.

Equations 2.35 and 2.36 also give

$$\begin{aligned}\tan \theta &= -\tan \phi \\ &= \tan (\pi - \phi) \\ \text{or, } \phi + \theta &= \pi\end{aligned}\tag{2.38}$$

Hence, equation 2.37 becomes

$$\xi = b' (\pi - \theta)\tag{2.39}$$

The equation 2.39 can be conveniently utilised to get the cam contour for a given follower motion during the rise period. Similarly set of equations 2.32 is used to get the corresponding follower surface.

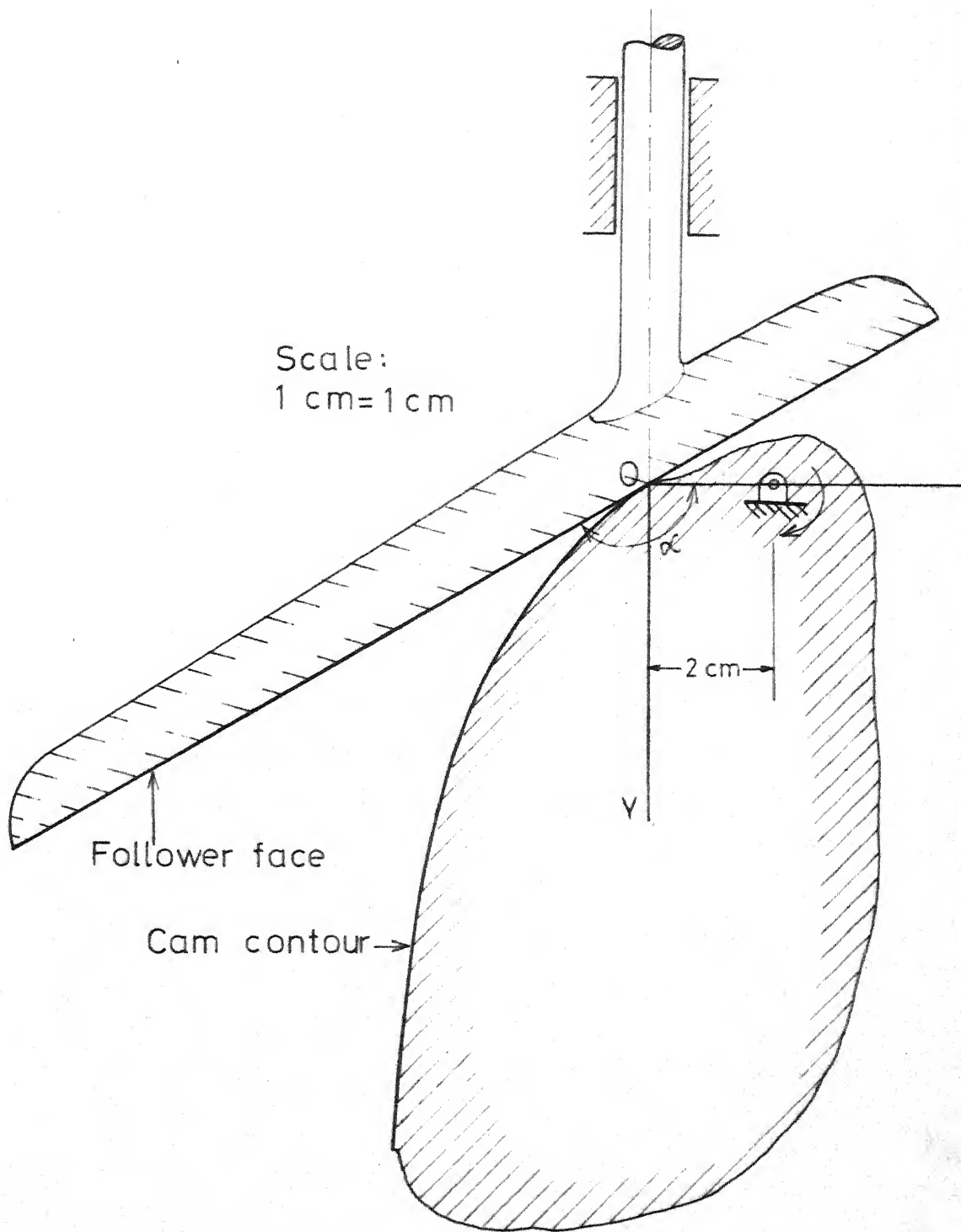
2.3 SOME ILLUSTRATIVE CASES

Now, for rolling contact the cam-contour and the corresponding follower surface will be developed for different cases, using the formulation discussed in Section 2.2. Some of the important cases are as follows:

Case I. Follower surface specified: Under this category some of the important cases are undermentioned.

A. Flat-face Follower:

Here, given the flat face follower, the corresponding cam contour is determined.



FLAT FACE FOLLOWER

FIG-2.5

Referring to the Fig. 2.5, it is assumed that

$$Y = m X \quad (2.40)$$

represents the follower-surface. m is the slope of the flat-face and it is a preassigned quantity.

Substituting equation 2.40 in the parametric equations 2.32 of the follower surface, one gets

$b = m (a_0 - b')$, which can be further written as

$$b' + \frac{1}{m} b = a_0 \quad (2.41)$$

This is a first order linear differential equation whose solution can be written as

$$b = A_1 e^{-\phi/m} + a_0 m \quad (2.42)$$

A_1 is an arbitrary constant whose value is to be determined using suitable boundary condition.

Taking $b = 0$ at $\phi = 0$, equation 2.42 gives

$$0 = A_1 + a_0 m, \text{ which gives } A_1 = -a_0 m \quad (2.43)$$

Equations 2.42 and 2.43 combine to give

$$b = a_0 m (1 - e^{-\phi/m}) \quad (2.44)$$

This gives the equation of the follower-motion in terms of angle of rotation of cam. Differentiating both sides of the equation 2.44, with respect to ϕ , it gives

$$b' = a_0 e^{-\phi/m} \quad (2.45)$$

Then, equation 2.37 becomes

$$\xi = a_o e^{-\phi/m} \quad (2.46)$$

But, $\phi = \pi - \theta$, as per equation 2.38

$$\text{Hence, } \xi = a_o e^{-\frac{1}{m}(\pi - \theta)}$$

$$\text{or } \xi = a_o e^{-\pi/m} e^{\theta/m}$$

$$\text{or } \xi = \xi_o e^{\theta/m}$$

$$\text{Where, } \xi_o = a_o e^{-\pi/m} = \text{a constant}$$

Equation 2.47 represents a logarithmic spiral.

Thus for pure rolling motion, corresponding to a flat-face follower the cam-contour is logarithmic spiral. The motion of the follower is an exponential function of ϕ .

B. Parabolic-face Follower

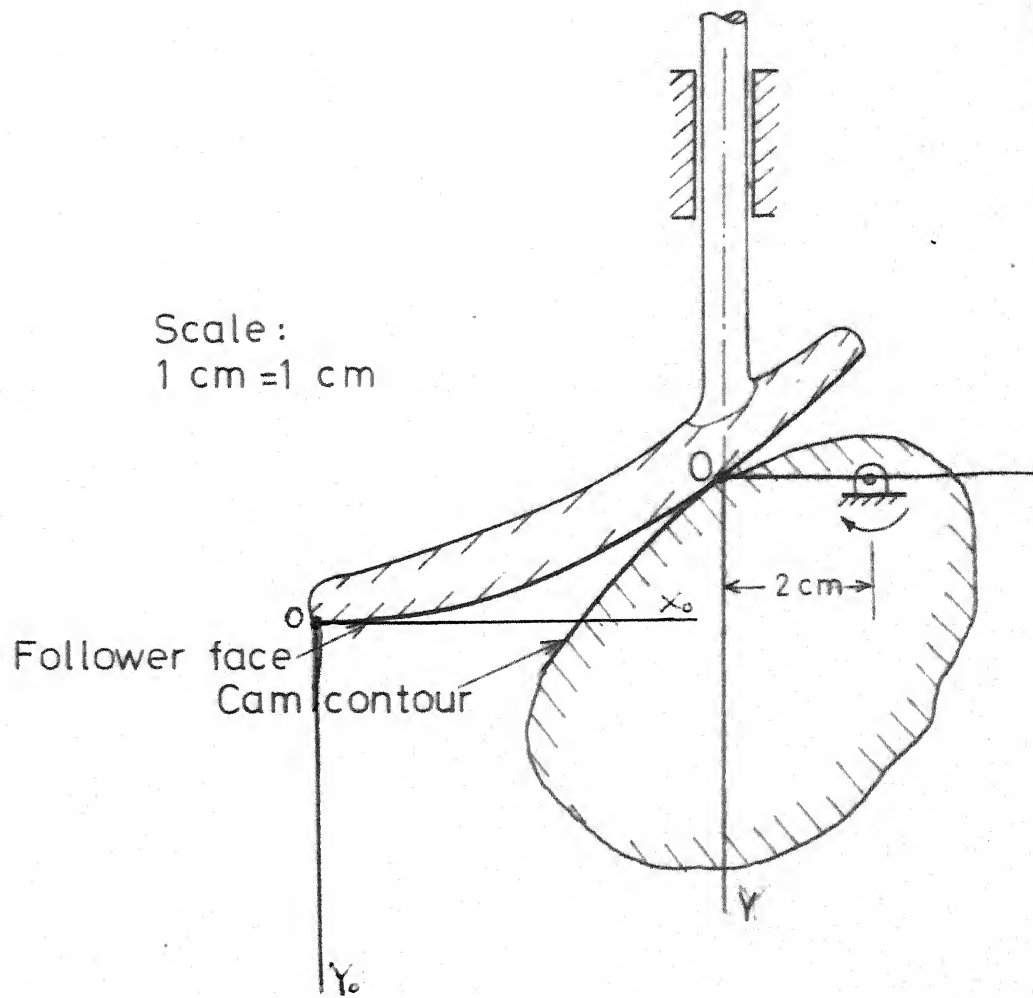
Here, the cam contour is determined given that the follower face is parabolic.

Referring to Fig. 2.6, the equation of the parabolic contour with respect to the co-ordinate-system $X_o O Y_o$ is given by

$$X_o^2 = -4p Y_o \quad (2.48)$$

where p is an assignable constant.

A new co-ordinate-system XOY, as shown in Fig. 2.6, is chosen. With respect to XOY, the equation of the parabolic contour can be written as



FOLLOWER FACE : PARABOLIC

FIG.2.6

$$\begin{aligned}
 (l_o + X)^2 &= -4 p (Y - m_o) \\
 \text{or } (l_o + X)^2 &= 4 p (m_o - Y)
 \end{aligned}
 \tag{2.49}$$

$(l_o, -m_o)$ being the co-ordinates of the origin of XOY and it is on the parabola only.

Hence, from equation 2.48,

$$\begin{aligned}
 l_o^2 &= -4 p (-m_o) \\
 \text{or } l_o^2 &= 4 p m_o
 \end{aligned}
 \tag{2.50}$$

But, Equations 2.32 give the parametric equation of the follower surface as

$$\begin{aligned}
 X &= a_o - b' \\
 Y &= b
 \end{aligned}
 \tag{2.51}$$

Substituting X and Y from (2.51) in (2.49), it gives

$$\begin{aligned}
 (l_o + a_o - b')^2 &= 4 p (m_o - b) \\
 \text{or } (l_o + a_o - b') &= 2 \sqrt{p} (m_o - b)^{1/2} \\
 \text{or } \frac{db}{d\theta} &= (l_o + a_o) - 2 \sqrt{p} (m_o - b)^{1/2}
 \end{aligned}$$

Putting $k_o = l_o + a_o$

and $d = 2 \sqrt{p}$, above equation can be written as

$$\begin{aligned}
 \frac{db}{d\theta} &= k_o - d (m_o - b)^{1/2} \\
 \text{or } \frac{db}{k_o - d (m_o - b)^{1/2}} &= d \theta
 \end{aligned}$$

Integration of both sides yields

$$\int \frac{db}{k_o - d (m_o - b)^{1/2}} = \theta + A_2,
 \tag{2.52}$$

A_2 is the constant of integration whose value is determined using the proper boundary condition.

After integrating the left hand side of (2.52), it becomes

$$\frac{2 k_o}{d^2} \left[\ln \left(1 - \frac{d}{k_o} \sqrt{m_o - b} \right) - \left(1 - \frac{d}{k_o} \sqrt{m_o - b} \right) \right] = \phi + A_2 \quad (2.53)$$

Taking $b = 0$ at $\phi = 0$, equation 2.53 gives

$$\frac{2 k_o}{d^2} \left[\ln \left(1 - \frac{d}{k_o} \sqrt{m_o} \right) - \left(1 - \frac{d}{k_o} \sqrt{m_o} \right) \right] = A_2 \quad (2.54)$$

Substituting the value of A_2 from (2.54) in (2.53), it becomes

$$\frac{2 k_o}{d^2} \left[\ln \frac{k_o - d \sqrt{m_o - b}}{k_o - d \sqrt{m_o}} + \frac{d}{k_o} \left(\sqrt{m_o - b} - \sqrt{m_o} \right) \right] = \phi \quad (2.55)$$

This equation gives the type of follower motion.

Now, as per equation 2.37, cam contour is given by

$$r = b' \quad (2.56)$$

$$\text{But } b' = k_o - d (m_o - b)^{1/2}$$

Hence

$$r = k_o - d (m_o - b)^{1/2} \quad (2.57)$$

Combining equations 2.55 and 2.57, it can be obtained that

$$\frac{2 k_o}{d^2} \left[\ln \frac{r}{k_o} - \frac{r}{k_o} \right] = \phi + \frac{2 k_o}{d^2} \left[\ln \frac{k_o - d \sqrt{m_o}}{k_o} - \frac{k_o - d \sqrt{m_o}}{k_o} \right] \quad (2.58)$$

This gives the cam contour for a follower with parabolic surface and hence the cam contour can be plotted.

Case 2. Specified Follower Motion

Here, for given follower motion, the cam contour as well as the follower surface is synthesised. Under this category some examples are illustrated below:

A. Uniform follower motion:

In this case, the follower displacement is given by the equation:

$$b = \frac{L_r}{\phi_r} \phi \quad (2.59)$$

L_r is the total lift of the follower during rolling contact and ϕ_r , the corresponding angle of cam rotation.

Now, as per equation 2.37, cam contour is given by

$$\xi = b'$$

which in this case becomes

$$\xi = \frac{L_r}{\phi_r}$$

or $\xi = \xi_0$, where $\xi_0 = \frac{L_r}{\phi_r} \quad (2.60)$

This is equation of a circle and thus in this case the cam contour is a circle.

Also, from equation 2.32, the follower surface is represented by the parametric equations

$$X = a_o - b'$$

and $Y = b$

which here become

$$\begin{aligned} X &= a_o - \epsilon_o = c_o \\ Y &= \epsilon_o \phi \end{aligned} \quad (2.61)$$

Under the condition specified earlier the equations 2.61 represent a vertical straight line. With this cam and follower configuration, motion transfer from cam to the follower is not possible. Hence uniform motion of the follower cannot be obtained with pure rolling action.

B. Follower displacement curve : parabolic

Here, the follower motion is given by

$$b = \frac{L_r}{\phi_r^2} \phi^2 \quad (2.62)$$

Differentiation of (2.62) with respect to ϕ gives

$$b' = \frac{2 L_r}{\phi_r^2} \phi \quad (2.63)$$

Hence, as per equation 2.37 cam contour is given by the equation,

$$\epsilon = \frac{2 L_r}{\phi_r^2} \phi \quad (2.64)$$

Similarly, the follower surface is represented by the parametric equations 2.32, which in this case becomes

$$\begin{aligned} X &= a_o - \frac{2 L_r}{\phi_r^2} \phi \\ \text{and } Y &= \frac{L_r}{\phi_r^2} \phi^2 \end{aligned} \quad (2.65)$$

Thus (2.64) gives the cam-contour and (2.65) give the corresponding follower surface.

C. Simple Harmonic Motion of the Follower:

In this case, the follower motion is given by

$$b = \frac{L_r}{2} \left(1 - \cos \frac{\pi}{\phi_r} \phi \right), \quad 0 \leq \phi \leq \phi_r \quad (2.66)$$

Differentiating it once with respect to ϕ , it gives

$$b' = \frac{\pi L_r}{2 \phi_r} \sin \frac{\pi \phi}{\phi_r} \quad (2.67)$$

Thus, as per equation 2.37, cam contour is given by

$$\xi = \frac{\pi L_r}{2 \phi_r} \sin \frac{\pi \phi}{\phi_r} \quad (2.68)$$

Similarly, the follower surface is represented by the parametric equation 2.32 which here becomes

$$X = a_o - \frac{\pi L_r}{2 \phi_r} \sin \frac{\pi \phi}{\phi_r} \quad (2.69)$$

$$\text{and } Y = \frac{L_r}{2} \left(1 - \cos \frac{\pi \phi}{\phi_r} \right)$$

Equation 2.68 gives the cam-contour and the equation 2.69 the follower surface.

D. Cycloidal Motion of the Follower:

Here, the follower motion is given by the relation

$$b = L_r \left(\frac{\phi}{\phi_r} - \frac{1}{2\pi} \sin \frac{2\pi \phi}{\phi_r} \right), \quad 0 < \phi \leq \phi_r \quad (2.70)$$

Differentiating it once with respect to ϕ ,
it gives

$$b' = \frac{L_r}{\phi_r} (1 - \cos \frac{2\pi\phi}{\phi_r}) \quad (2.71)$$

Combining equations 2.37 and 2.71 the cam contour is given by

$$\xi = \frac{L_r}{\phi_r} (1 - \cos \frac{2\pi\phi}{\phi_r}) \quad (2.72)$$

And the follower surface is represented by the set of equations 2.32 which in this case becomes

$$\begin{aligned} X &= a_o - \frac{L_r}{\phi_r} (1 - \cos \frac{2\pi\phi}{\phi_r}) \\ \text{or } X &= (a_o - \frac{L_r}{\phi_r}) + \frac{L_r}{\phi_r} \cos \frac{2\pi\phi}{\phi_r} \\ \text{and } Y &= L_r (\frac{\phi}{\phi_r} - \frac{1}{2\pi} \sin \frac{2\pi\phi}{\phi_r}) \end{aligned} \quad (2.73)$$

Equation 2.72 gives the cam contour and the set of equations 2.73 represent the follower surface.

E. Any arbitrary type of follower motion:

If the lifts of the follower for various angles of cam rotation are given, the cam contour and the corresponding follower surface can be determined using the equations 2.37 and 2.32.

Under the present work, for sets of values, cam contours and the corresponding follower surfaces are determined, using the computational methods.

2.4 LIMITATIONS

For the transfer of motion to the follower with pure rolling contact action, the rise curve must have the increasing slope throughout. This implies that b'' must be positive during the rise of the follower.

This is a primary criteria and it must be fulfilled by each displacement diagram, otherwise the motion cannot be transferred to the follower.

Now, rewriting the equation 2.32

$$X = a_0 - b' \quad \text{and} \quad Y = b$$

Above equations on differentiation can give

$$\frac{dX}{dY} = - \frac{b''}{b'} \quad (2.74)$$

But b' and b'' both have to be positive, therefore,

$\frac{dX}{dY}$ must be negative. This condition must be satisfied by the follower surface. It is to be noted that the primary limitation is that b' has to be increasing throughout the rise-period.

Another limitation is that the follower motion should be so selected that the pressure angle should not where exceed a maximum allowable value. In all the numerical examples the maximum allowable pressure angle has been assumed to be 30° .

CHAPTER 3

FORCE ANALYSIS AND WORK-LOSS ESTIMATION DURING ROLLING CONTACT

In this chapter, the forces acting on the cam follower system during rolling contact have been analysed using the equilibrium equations. The reactive forces in the guide are evaluated which in turn are used to estimate the work-loss in the guide due to friction.

3.1 FORCE ANALYSIS OF THE CAM FOLLOWER SYSTEM

In the case of pure rolling contact between cam and follower, the instantaneous pressure angle ' α ' is given by, (Fig. 3.1)

$$\tan (\pi - \alpha) = \frac{dY}{dX} \quad (3.1)$$

$$\text{or } \tan \alpha = - \frac{dY}{dX} \quad (3.2)$$

But, equation 2.32 gives

$$X = a_0 - b' \quad (3.3)$$

$$\text{and } Y = b \quad (3.4)$$

which can be differentiated to obtain

$$\frac{dY}{dX} = - \frac{b'}{b''} \quad (3.5)$$

combining equations 3.2 and 3.5 yields

$$\tan \alpha = \frac{b'}{b''} \quad (3.6)$$

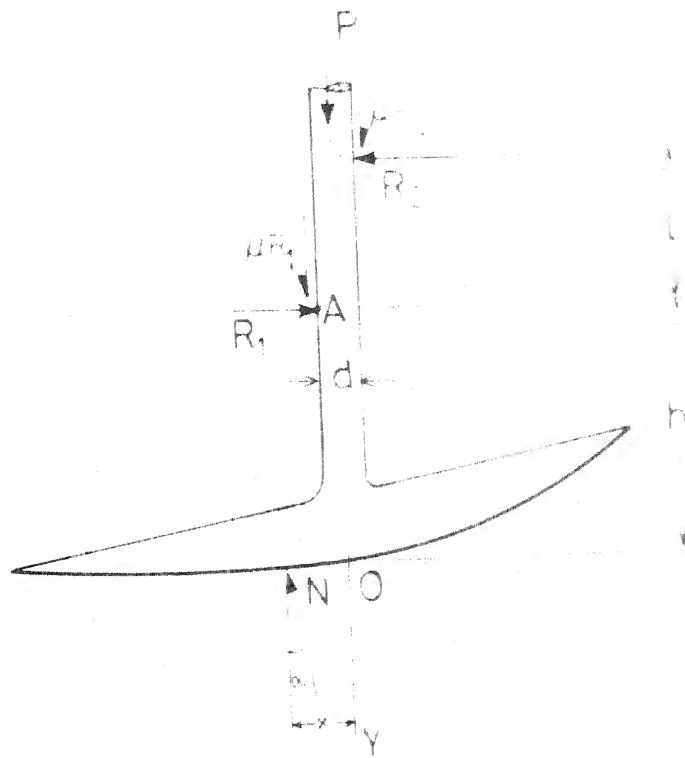


FIG. 3.1

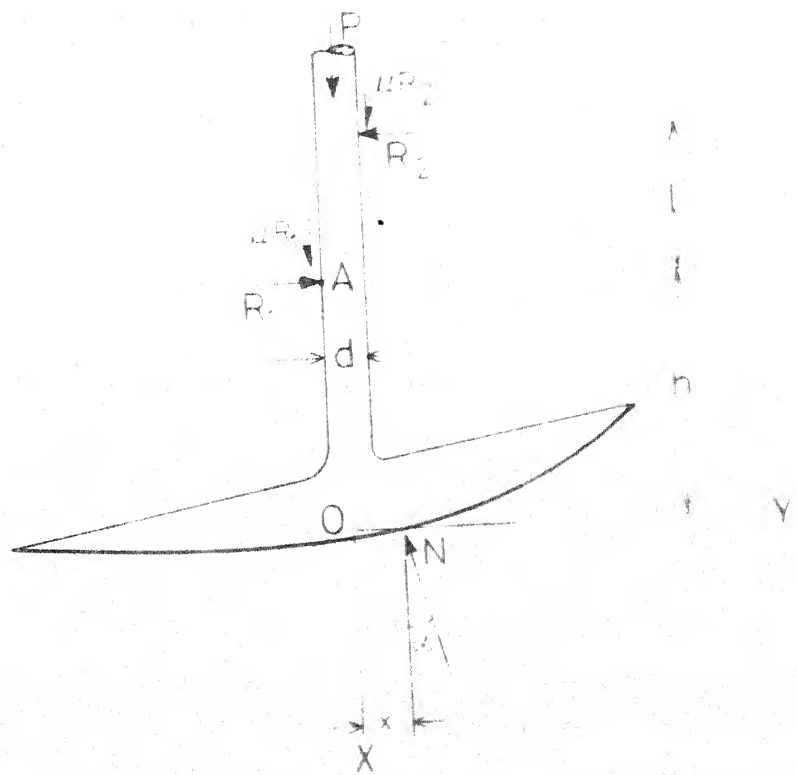


FIG. 3.2

Now, for the force analysis of the cam follower system, different cases arise depending upon the value of X . They are:

Case I : $X \leq 0$,

Case II : $0 < X \leq h \tan \alpha$,

Case III : $h \tan \alpha < X \leq (h + 1) \tan \alpha$

and Case IV : $X > (h + 1) \tan \alpha$.

Now, different cases will be dealt separately and in each case jamming forces R_1 and R_2 will be determined to evaluate the work loss in the guide.

Case I : $X \leq 0$

Referring to Fig. 3.1, considering the equilibrium of forces on the follower, along horizontal - axis

$$R_1 = R_2 + N \sin \alpha \quad (3.7)$$

Similarly, equilibrium of vertical components of forces gives

$$N \cos \alpha = P + \mu (R_1 + R_2) \quad (3.8)$$

where $P = P_0 + k \cdot b$, is the compressive force on the guide.

And the moment of forces along the point A, for equilibrium gives

$$\begin{aligned} R_2 \times l &= X (N \cos \alpha) + h (N \sin \alpha) \\ \text{or } R_2 &= \frac{N}{l} (X \cos \alpha + h \sin \alpha) \end{aligned} \quad (3.9)$$

Now eliminating R_1 from equations 3.7 and 3.8, it gives

$$R_2 = \frac{1}{2} \left[\frac{N \cos \alpha - P}{\mu} - N \sin \alpha \right] \quad (3.10)$$

Hence, from (3.9) and (3.10)

$$\frac{N}{1} (X \cos \alpha + h \sin \alpha) = \frac{1}{2} \left[\frac{N \cos \alpha - P}{\mu} - N \sin \alpha \right]$$

which can be simplified to give

$$N = \frac{P \cdot 1}{(1 - 2\mu X) \cos \alpha - \mu (2h + 1) \sin \alpha} \quad (3.11)$$

Substituting N from (3.11) in (3.9), it gives

$$R_2 = \frac{P (X + h \tan \alpha)}{(1 - 2\mu X) - \mu (2h + 1) \tan \alpha} \quad (3.12)$$

Hence, equation 3.7 becomes

$$R_1 = \frac{P [X + (h + 1) \tan \alpha]}{(1 - 2\mu X) - \mu (2h + 1) \tan \alpha} \quad (3.13)$$

Now, from equation 3.11, to avoid locking of the follower in the guide, 'N' must always be finite, which implies the denominator of (3.11) must never be zero.

$(1 - 2\mu X) \cos \alpha - \mu (2h + 1) \sin \alpha > 0$, for the present purpose.

This simplifies to give

$$1 > \frac{2\mu (X + h \tan \alpha)}{1 - \mu \tan \alpha} \quad (3.14)$$

Case II: $0 < X \leq h \tan \alpha$

Referring to Fig. 3.2, considering the equilibrium of forces on the follower, along horizontal direction

$$R_1 = R_2 + N \sin \alpha \quad (3.15)$$

Similarly, the equilibrium of vertical components of forces on the follower gives

$$N \cos \alpha = P + \mu (R_1 + R_2) \quad (3.16)$$

And, the moment of forces on the follower about the point A, for equilibrium demands

$$\begin{aligned} R_2 \times l &= h (N \sin \alpha) - X (N \cos \alpha), \text{ which gives} \\ R_2 &= \frac{N}{l} (h \sin \alpha - X \cos \alpha) \end{aligned} \quad (3.17)$$

Eliminating R_1 from equations 3.15 and 3.16, it gives

$$R_2 = \frac{1}{2} \left[\frac{N \cos \alpha - P}{\mu} - N \sin \alpha \right] \quad (3.18)$$

Hence, (2.17) and (2.18) give

$$\frac{N}{l} (h \sin \alpha - X \cos \alpha) = \frac{1}{2} \left[\frac{N \cos \alpha - P}{\mu} - N \sin \alpha \right]$$

which simplifies to give

$$N = \frac{P \cdot l}{(2\mu X + 1) \cos \alpha - \mu (2h + 1) \sin \alpha} \quad (3.19)$$

Substituting for N from (3.19) into (3.17), it becomes

$$R_2 = \frac{P (h \tan \alpha - X)}{(2\mu X + 1) - \mu (2h + 1) \tan \alpha} \quad (3.20)$$

Hence, (2.15) gives

$$R_1 = \frac{P [(h + 1) \tan \alpha - X]}{(2\mu X + 1) - \mu (2h + 1) \tan \alpha} \quad (3.21)$$

In the similar manner as discussed for case I, to avoid locking of the follower, from (3.19)

$$(2\mu X + 1) \cos \alpha - (2h + 1) \sin \alpha > 0$$

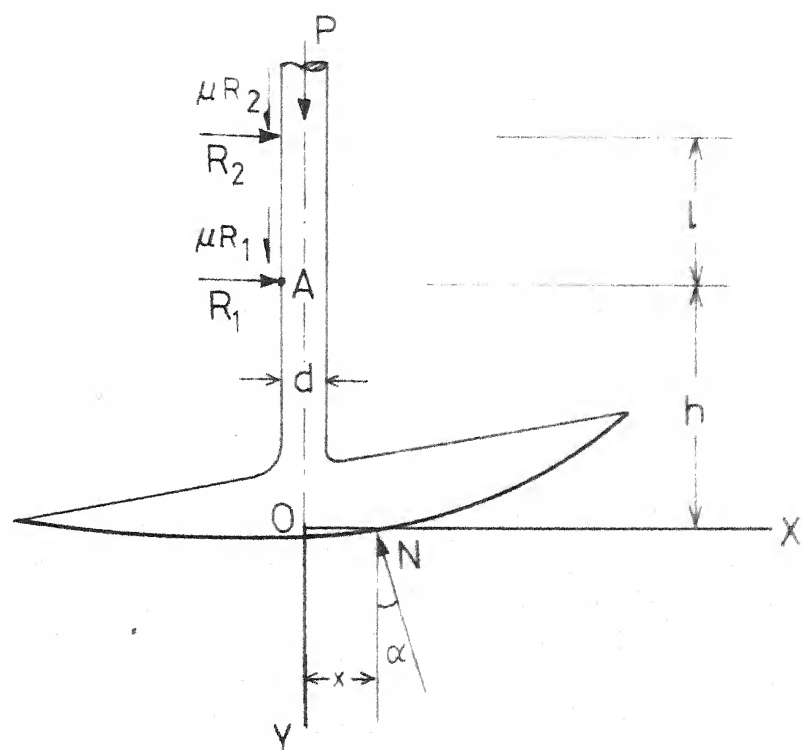


FIG. 3-3

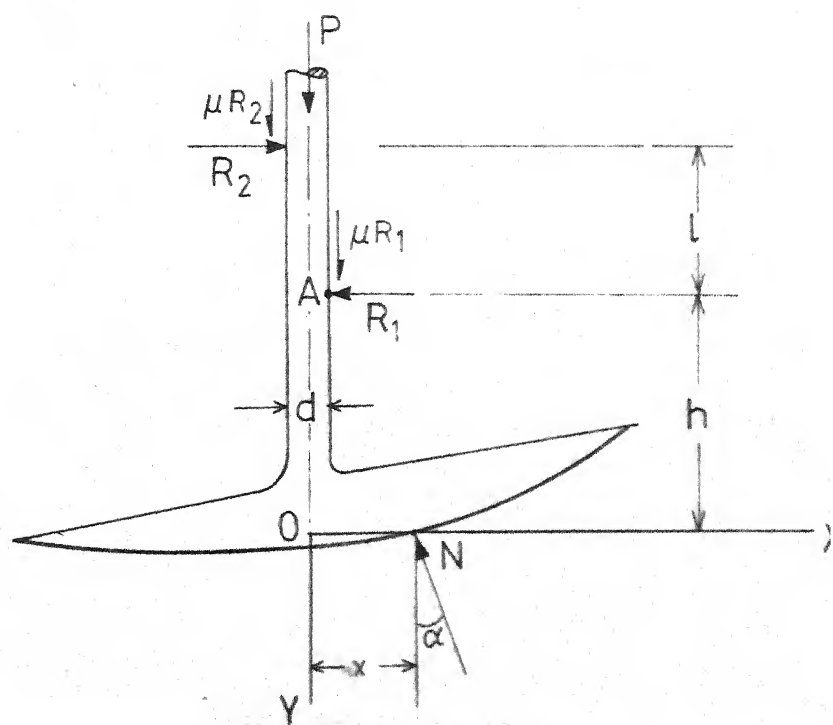


FIG. 3-4

which simplifies to give

$$1 > \frac{2\mu(h \tan \alpha - X)}{1 - \mu \tan \alpha} \quad (3.22)$$

Case III : $h \tan \alpha < X \leq (h + 1) \tan \alpha$

Referring to Fig. 3.3, the equilibrium of forces on the follower, along horizontal-direction gives

$$R_1 + R_2 = N \sin \alpha \quad (3.23)$$

Similarly, the equilibrium of forces on the follower, along vertical direction gives

$$N \cos \alpha = P + \mu(R_1 + R_2) \quad (3.24)$$

And, the moment of forces on the follower about the point A gives

$$R_2 \times l = N (X \cos \alpha - h \sin \alpha)$$

$$\text{or } R_2 = \frac{N}{l} (X \cos \alpha - h \sin \alpha) \quad (3.25)$$

Now, combining (3.23) and (3.24), it gives

$$N = \frac{P}{\cos \alpha - \mu \sin \alpha} \quad (3.26)$$

Substituting N from (3.26) into (3.25), it gives

$$R_2 = \frac{P (X - h \tan \alpha)}{l (1 - \mu \tan \alpha)} \quad (3.27)$$

Hence, equation 2.23 gives

$$R_1 = \frac{P [(1 + h) \tan \alpha - X]}{l (1 - \mu \tan \alpha)} \quad (3.28)$$

In the similar manner as discussed for case I, to avoid locking of the follower, from equation 3.19 the pertinent condition is

$$\cos \alpha - \mu \sin \alpha > 0$$

which gives

$$\tan \alpha < \frac{1}{\mu} \quad (3.29)$$

Case IV: When $X > (h + 1) \tan \alpha$

Referring to the Fig. 3.4, the equilibrium of forces acting on the follower, along horizontal direction demands

$$R_2 = R_1 + N \sin \alpha \quad (3.30)$$

Similarly, for the equilibrium of forces acting on the follower, the equation for the vertical direction is

$$N \cos \alpha = P + \mu (R_1 + R_2) \quad (3.31)$$

And the moment of forces about the point A, for equilibrium requires

$$R_2 \times l = X (N \cos \alpha) - h (N \sin \alpha)$$

$$\text{or } R_2 = \frac{N}{l} (X \cos \alpha - h \sin \alpha) \quad (3.32)$$

Eliminating R_1 from equations 3.30 and 3.31,

$$R_2 = \frac{1}{2} \left[\frac{N \cos \alpha - P}{\mu} + N \sin \alpha \right] \quad (3.33)$$

Combining (3.32) and (3.33) yields

$$\frac{N}{l} (X \cos \alpha - h \sin \alpha) = \frac{1}{2} \left[\frac{N \cos \alpha - P}{\mu} + N \sin \alpha \right]$$

which simplifies to give

$$N = \frac{P \cdot 1}{\mu(2h + 1) \sin \alpha - (2\mu X - 1) \cos \alpha} \quad (3.34)$$

Substituting for N from (3.34) into (3.32) yields

$$R_2 = \frac{P(X - h \tan \alpha)}{\mu(2h + 1) \tan \alpha - (2\mu X - 1)} \quad (3.35)$$

And hence equation 3.30 gives

$$R_1 = \frac{P[X - (h + 1) \tan \alpha]}{\mu(2h + 1) \tan \alpha - (2\mu X - 1)} \quad (3.36)$$

As discussed in the Case I, the condition to avoid locking of the follower is given from the equation 3.34 as

$$\mu(2h + 1) \sin \alpha - (2\mu X - 1) \cos \alpha > 0$$

which gives

$$1 > \frac{2\mu(X - h \tan \alpha)}{1 + \mu \tan \alpha} \quad (3.37)$$

3.2 WORK LOSS IN THE GUIDE

Work loss due to friction in the guide during the rise of the follower is given by

$$W_{gr} = \int_0^{L_r} \mu (R_1 + R_2) db \quad (3.38)$$

$$\text{But } db = b' d\theta$$

Hence (3.38) becomes

$$W_{gr} = \int_0^{\theta_r} \mu (R_1 + R_2) b' d\theta \quad (3.38)$$

Values of R_1 and R_2 are suitably taken from the different cases discussed earlier and work-loss is evaluated. This work loss gives the estimation of friction present in the

CHAPTER 4

CAM FOLLOWER SYNTHESIS WHEN SLIDING IS PRESENT

It has been established that for pure rolling contact between the cam and the follower surfaces, b' must be increasing throughout the rise period. But in case of dwell-rise-dwell cam this is not practicable and hence to do away with this difficulty the total lift of the follower is divided into two parts: the first part provides the rolling action and the second part permits sliding also.

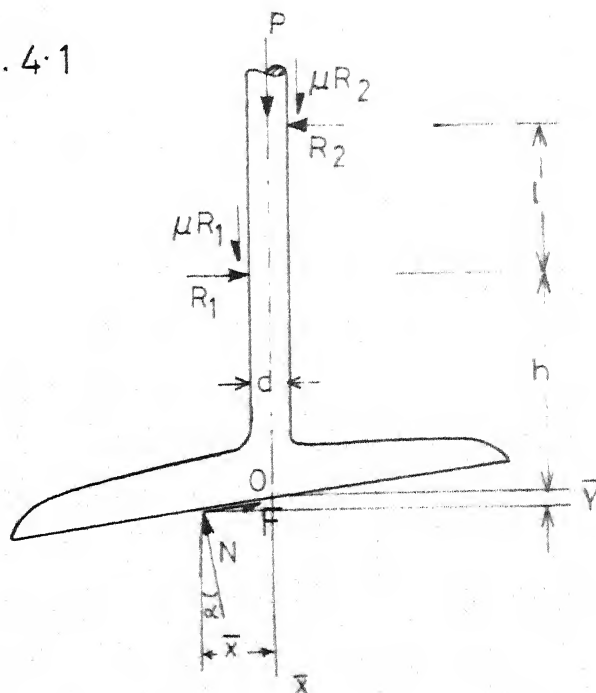
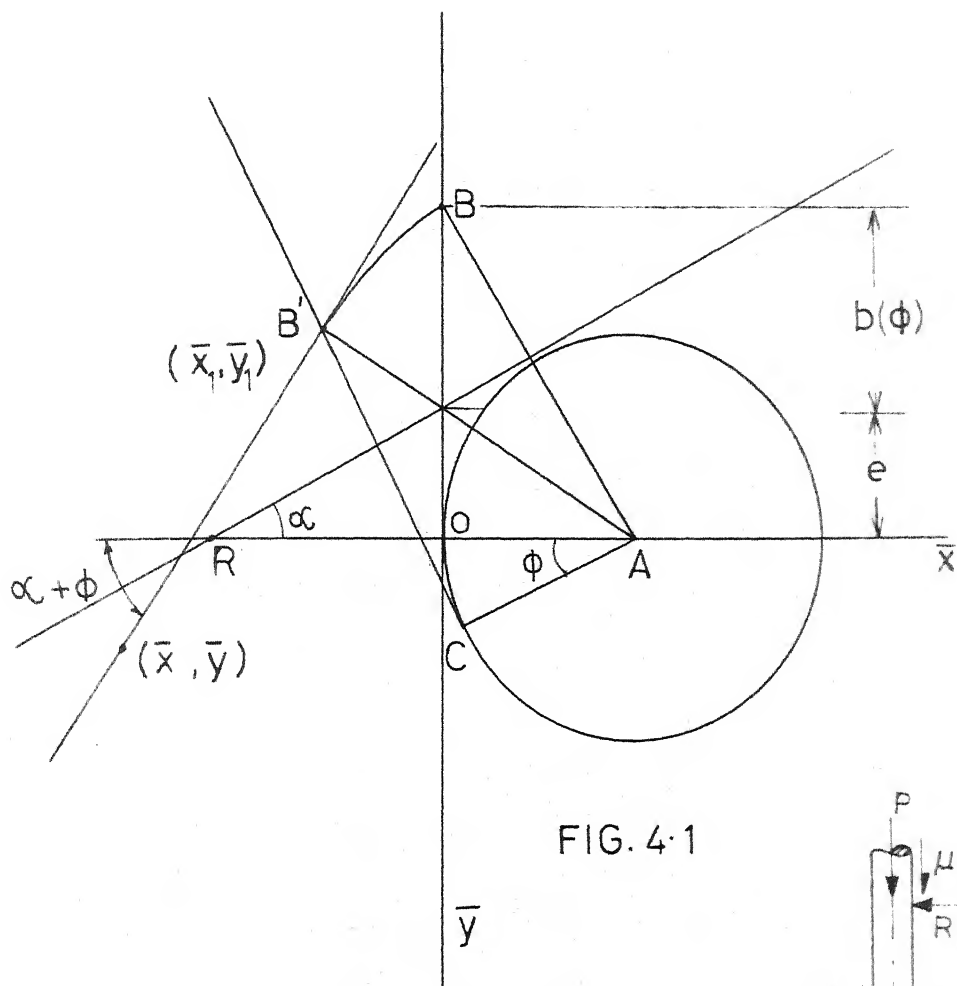
The synthesis of cam-follower system for pure rolling motion has already been given in Chapters 2 and 3. Here the synthesis for the sliding part has been discussed.

4.1 CAM CONTOUR SYNTHESIS WHEN SLIDING IS ALLOWED BETWEEN THE CONTACT SURFACES

Here it is assumed that the follower surface is flat.

The similar analysis is present in the book [2] for flat-face horizontal follower, but in the present situation the follower face is inclined to the horizontal (Fig. 4.1) and hence the analysis requires some modifications.

The basic principle of synthesis is as follows: The cam is held fixed and the follower is rotated at the cam speed in a direction opposite to the cam. The co-ordinates of the trace-point are found out as function of



θ in convenient co-ordinate system. Then equation for the family of curves representing the follower surface is obtained. Finally the envelope of the family of curves is determined to represent the cam-contour.

Referring to the Fig. 4.1, the point R represents the point on the follower surface at the end of rolling action. Beyond this the follower surface is assumed to be flat-face having an inclination ' α ' with the horizontal axis.

Co-ordinate system $\bar{x}\bar{y}$ is chosen as shown in Fig. 4.1. Using the principle of inversion, the follower position corresponding to angle of rotation θ is shown in Fig. 4.1. Referring to the Fig. 4.1, in triangles AOB and ACB'

$$AB = AB',$$

$$AO = AC$$

$$\text{and } \angle AOB = \angle ACB' = 90^\circ$$

Therefore, both the triangles are congruent

Thus,

$$\begin{aligned} CB' &= OB \\ &= e + b \end{aligned}$$

B' is the trace point whose co-ordinates are, (Fig. 4.1)

$$\bar{x}_1 = - [a_o \cos \theta + (e + b) \sin \theta - a_o] \quad (4.1)$$

$$\text{and } \bar{y}_1 = - [(e + b) \cos \theta - a_o \sin \theta] \quad (4.2)$$

The slope of the line representing the flat face at this position is given by

$$m = \tan [\pi - (\alpha + \phi)]$$

or $m = -\tan (\alpha + \phi)$ (4.3)

Hence the equation of the line will be

$$\bar{y} - \bar{y}_1 = m (\bar{x} - \bar{x}_1) \quad (4.4)$$

which on substitution of m from (4.3) in (4.4) becomes

$$\bar{y} - \bar{y}_1 = -\tan (\alpha + \phi) (\bar{x} - \bar{x}_1)$$

or $(\bar{y} - \bar{y}_1) \cos (\alpha + \phi) + (\bar{x} - \bar{x}_1) \sin (\alpha + \phi) = 0$ (4.5)

Substituting for \bar{x}_1 and \bar{y}_1 from (4.1) and (4.2) respectively into (4.5), it simplifies to give

$$\begin{aligned} \bar{y} \cos (\alpha + \phi) + \bar{x} \sin (\alpha + \phi) + (e + b) \cos \alpha \\ + a_o \sin \alpha - a_o \sin (\alpha + \phi) = 0 \end{aligned} \quad (4.6)$$

This represents the family of curves representing the follower surface.

To get the envelope, the equation 4.6 is differentiated with respect to ϕ , it gives

$$\begin{aligned} -\bar{y} \sin (\alpha + \phi) + \bar{x} \cos (\alpha + \phi) + b' \cos \alpha \\ - a_o \cos (\alpha + \phi) = 0 \end{aligned} \quad (4.7)$$

Equations 4.6 and 4.7 can be solved to give

$$\begin{aligned} \bar{x} = - \left[(e + b) \cos \alpha \sin (\alpha + \phi) + b' \cos \alpha \cos (\alpha + \phi) \right. \\ \left. + a_o \sin \alpha \sin (\alpha + \phi) - a_o \right] \end{aligned} \quad (4.8)$$

and

$$\bar{y} = - \left[(e + b) \cos \alpha \cos (\alpha + \phi) - b' \cos \alpha \sin (\alpha + \phi) + a_0 \sin \alpha \cos (\alpha + \phi) \right] \quad (4.9)$$

The set of equations 4.8 and 4.9 are the parametric equations of the cam contour for the sliding portion and thus they are used to get the cam contour.

4.2 CONDITION TO AVOID CUSP FORMATION IN THE CAM CONTOUR

For the flat face follower, the cam profile must be convex everywhere. To ensure this there should not be any cusp formation in the cam contour.

The condition for cusp formation is given by

$$\frac{d \bar{x}}{d \phi} = \frac{d \bar{y}}{d \phi} = 0 \quad (4.10)$$

But equations 4.8 and 4.9 give

$$\frac{d \bar{x}}{d \phi} = - \left[(e + b + b'') \cos \alpha + a_0 \sin \alpha \right] \cos (\alpha + \phi) \quad (4.11)$$

$$\text{and } \frac{d \bar{y}}{d \phi} = \left[(e + b + b'') \cos \alpha + a_0 \sin \alpha \right] \sin (\alpha + \phi) \quad (4.12)$$

Thus to avoid cusp formation, as per (4.10)

$$(e + b + b'') \cos \alpha + a_0 \sin \alpha > 0 \quad (4.13)$$

This condition must be satisfied for all values of ϕ to avoid any cusp formation in the cam contour.

4.3 MATCHING CONDITIONS FOR ROLLING AND SLIDING PARTS

It is assumed that ξ_1 be the radial distance of the cam contour corresponding to θ_r (which signifies the end of rolling part.)

For the sliding part, for $\theta = 0$ equation 4.8 gives

$$\bar{x}_0 = \bar{x} \text{ at } \theta = 0$$

$$\text{i.e., } \bar{x}_0 = - \left[e \cos \alpha \sin \alpha + b'_0 \cos^2 \alpha + a_0 \sin^2 \alpha - a_0 \right] \quad (4.14)$$

where $b'_0 = b'$ at $\theta = 0$, which is taken to be ξ_0

But matching condition demands

$$\bar{x}_0 = - (\xi_1 - a_0) \quad (4.15)$$

Thus, from (4.14) and (4.15) it gives

$$- (\xi_1 - a_0) = - \left[e \cos \alpha \sin \alpha + b'_0 \cos^2 \alpha + a_0 \sin^2 \alpha - a_0 \right]$$

which simplifies to

$$e \cos \alpha \sin \alpha + \xi_0 \cos^2 \alpha + a_0 \sin^2 \alpha = \xi_1 \quad (4.16)$$

Other matching condition is that \bar{y} at $\theta = 0$ should be zero.

This gives

$$- \left[e \cos^2 \alpha - \xi_0 \cos \alpha \sin \alpha + a_0 \sin \alpha \cos \alpha \right] = 0$$

which simplifies to give

$$e \cos \alpha = (\xi_0 - a_0) \sin \alpha \quad (4.17)$$

Combining (4.16) and (4.17), one gets

$$\xi_0 = \xi_1 \quad (4.18)$$

Then equation 4.17 becomes

$$e = (\rho_1 - a_o) \tan \alpha \quad (4.19)$$

Therefore, matching conditions are

$$\rho_o = \rho_1 \quad (4.20)$$

$$\text{and } e = (\rho_1 - a_o) \tan \alpha \quad (4.21)$$

Thus equations 4.20 and 4.21 provide the matching criteria for two portions, viz. rolling and sliding.

4.4 FORCE ANALYSIS OF THE CAM FOLLOWER SYSTEM DURING SLIDING

Here also the forces acting on the cam follower system are analysed. The friction force at the cam and follower interface is calculated which is further used to estimate the interface work loss. The reactive forces in the guide are determined which in turn are used for the estimation of work losses in the guide due to friction.

In this case, ' α ' the inclination of the follower face to the horizontal axis gives the pressure angle.

Referring to Fig. 4.2, the equilibrium equation for the forces acting on the follower, along horizontal direction gives

$$R_1 - R_2 - N \sin \alpha + F \cos \alpha = 0 \quad (4.22)$$

But the frictional force is given by

$$F = \mu_1 N \quad (4.23)$$

Hence, equation 4.22 becomes

$$R_1 - R_2 - N (\sin \alpha - \mu_1 \cos \alpha) = 0 \quad (4.24)$$

Similarly, for equilibrium of forces along vertical direction the equation is

$$P + \mu (R_1 + R_2) = N \cos \alpha + F \sin \alpha$$

Using (4.23) it becomes

$$P + \mu (R_1 + R_2) = N (\cos \alpha + \mu_1 \sin \alpha) \quad (4.25)$$

Eliminating R_1 between (4.24) and (4.25), it gives

$$R_2 = \frac{1}{2} \left[\frac{N (\cos \alpha + \mu_1 \sin \alpha) - P}{\mu} - N (\sin \alpha - \mu_1 \cos \alpha) \right] \quad (4.26)$$

Again, for the equilibrium, the moment of the forces acting on the follower about the point A gives

$$R_2 \times l - (F \sin \alpha + N \cos \alpha) \bar{x} + (F \cos \alpha - N \sin \alpha) (h + \bar{y}) = 0$$

which gives

$$R_2 = \frac{N}{l} \left[\bar{x} (\cos \alpha + \mu_1 \sin \alpha) + (\sin \alpha - \mu_1 \cos \alpha) (h + \bar{y}) \right] \quad (4.27)$$

Combining (4.26) and (4.27), and simplifying, it gives

$$N = \frac{P l}{(1 - 2\mu \bar{x})(\cos \alpha + \mu_1 \sin \alpha) - \mu(2h + 2\bar{y} + l)(\sin \alpha - \mu_1 \cos \alpha)} \quad (4.28)$$

Thus, by substituting N from (4.28) into (4.27) it gives

$$R_2 = \frac{P[\bar{x}(\cos \alpha + \mu_1 \sin \alpha) + (\sin \alpha - \mu_1 \cos \alpha)(h + \bar{y})]}{(1 - 2\mu\bar{x})(\cos \alpha + \mu_1 \sin \alpha) - \mu(2h + 2\bar{y} + 1)(\sin \alpha - \mu_1 \cos \alpha)} \quad (4.29)$$

From equations 4.24 and 4.29, R_1 can be obtained as

$$R_1 = \frac{P[\bar{x}(\cos \alpha + \mu_1 \sin \alpha) + (\sin \alpha - \mu_1 \cos \alpha)(h + \bar{y} + 1)]}{(1 - 2\mu\bar{x})(\cos \alpha + \mu_1 \sin \alpha) - \mu(2h + 2\bar{y} + 1)(\sin \alpha - \mu_1 \cos \alpha)} \quad (4.30)$$

In equations 4.29 and 4.30, the values of \bar{x} and \bar{y} are inserted from the relations 4.8 and 4.9. Above analysis is valid for \bar{x} less than zero, which is encountered in the present problem.

4.5 WORK LOSS IN THE GUIDE

Due to friction in the guide, the work loss during rise of the follower with no rolling contact action is given by

$$W_{gs} = \int_0^{L_s} \mu(R_1 + R_2) db \quad (4.31)$$

But $db = b' d\theta$, hence equation 4.31 becomes

$$W_{gs} = \int_0^{\theta_s} \mu(R_1 + R_2) b' d\theta \quad (4.32)$$

Values of R_1 and R_2 are put into (4.32) from equations 4.29 and 4.30.

4.6 WORK LOSS AT THE CAM AND FOLLOWER INTERFACE

Due to friction at the cam and follower interface, the loss of work is given by the equation

$$W_{fi} = \int_0^{\theta_s} F \sqrt{\left(\frac{d\bar{y}}{d\theta}\right)^2 + \left(\frac{d\bar{x}}{d\theta}\right)^2} d\theta \quad (4.33)$$

Now, squaring (4.11) and then adding it gives

$$\left(\frac{d\bar{x}}{d\theta}\right)^2 + \left(\frac{d\bar{y}}{d\theta}\right)^2 = \left[(e + b + b'') \cos \alpha + a_o \sin \alpha\right]^2$$

or

$$\sqrt{\left(\frac{d\bar{x}}{d\theta}\right)^2 + \left(\frac{d\bar{y}}{d\theta}\right)^2} = (e + b + b'') \cos \alpha + a_o \sin \alpha \quad (4.34)$$

From equations 4.23 and 4.28 it becomes

$$F = \frac{\mu_1 P l}{(1 - 2\mu\bar{x})(\cos \alpha + \mu_1 \sin \alpha) - \mu(2h + 2\bar{y} + 1)(\sin \alpha - \mu_1 \cos \alpha)} \quad (4.35)$$

Thus, using (4.34) and (4.35), equation 4.33 can be written as

$$W_{fi} = \int_0^{\theta_s} \frac{\mu_1 P l \left[(e + b + b'') \cos \alpha + a_o \sin \alpha\right] d\theta}{(1 - 2\mu\bar{x})(\cos \alpha + \mu_1 \sin \alpha) - \mu(2h + 2\bar{y} + 1)(\sin \alpha - \mu_1 \cos \alpha)} \quad (4.36)$$

The equation 4.36 gives the work loss at the cam-follower interface.

Thus, the interface work loss is evaluated using the equation 4.32 and the use of (4.36) is made to estimate the work loss in the guide. The estimation of these work-loss are important for design point of view.

CHAPTER 5

SYNTHESIS OF CAM FOLLOWER SYSTEM FOR THE RISE PERIOD OF A DWELL-RISE-DWELL-RETURN CAM AN EXAMPLE

It has been discussed in the Chapter 4 that pure rolling action between the cam surface and the follower surface is not possible for the entire rise period of the Dwell-Rise-Dwell cam. Therefore the rise of the follower is contributed initially by the rolling action and finally sliding is also permitted to occur between the contact surfaces. Thus cam synthesis for the rise period had to be done in two parts, part one "with rolling action" and part two "when sliding is permitted".

If L is the total lift of the follower for an angle of rotation ϕ_o of the cam, then it is assumed that lift L_r is contributed by rolling action (corresponding angle of cam rotation is ϕ_r) and the remaining lift ' L_s ' is contributed by sliding (corresponding angle of rotation is ϕ_s)

Obviously,

$$L = L_r + L_s \quad (5.1)$$

$$\text{and } \phi_o = \phi_r + \phi_s \quad (5.2)$$

5.1 CAM FOLLOWER SYNTHESIS WHEN SLIDING EXISTS

The follower motion is taken to be represented by the following equation

$$b = L_s \sin \frac{\pi \phi}{2 \phi_s} \quad (5.3)$$

Differentiating it once with respect to \emptyset ,

$$b' = \frac{\pi}{2} \frac{L_s}{\emptyset_s} \cos \frac{\pi \emptyset}{2 \emptyset_s} \quad (5.4)$$

From equation 4.20, the matching condition demands

$$b' \text{ at } \emptyset = 0 = \xi_1$$

Therefore, from (5.4)

$$\xi_1 = \frac{\pi}{2} \frac{L_s}{\emptyset_s} \quad (5.5)$$

From equation 4.13, to avoid cusp formation the condition is

$$(e + b + b'') + a_0 \tan \alpha > 0 \quad (5.6)$$

But from (4.21), $e = (\xi_1 - a_0) \tan \alpha$

Hence equation 5.6 gives

$$b + b'' + \xi_1 \tan \alpha > 0 \quad (5.7)$$

Here,

$$b'' = L_s \left(\frac{\pi}{2 \emptyset_s} \right)^2 \sin \frac{\pi \emptyset}{2 \emptyset_s}, \text{ from (5.4)} \quad (5.8)$$

Substituting the expression for b and b'' into equation 5.7, it gives

$$L_s \sin \frac{\pi \emptyset}{2 \emptyset_s} - L_s \left(\frac{\pi}{2 \emptyset_s} \right)^2 \sin \frac{\pi \emptyset}{2 \emptyset_s} + \xi_1 \tan \alpha > 0 \quad (5.9)$$

The relation 5.9 will be true for all \emptyset , if it is true for $\emptyset = \emptyset_s$.

$$\text{i.e., } L_s - L_s \left(\frac{\pi}{2 \emptyset_s} \right)^2 + \xi_1 \tan \alpha > 0$$

$$\text{or } \xi_1 \tan \alpha > L_s \left[\left(\frac{\pi}{2 \emptyset_s} \right)^2 - 1 \right] \quad (5.10)$$

Substituting the value of ϕ_1 from equation 5.5 into equation 5.10, it gives

$$\frac{\pi}{2} \frac{L_s}{\phi_s} \tan \alpha > L_s \left[\left(\frac{\pi}{2 \phi_s} \right)^2 - 1 \right]$$

which simplifies to give

$$\tan \alpha > \left(\frac{\pi}{2 \phi_s} - \frac{2}{\pi} \phi_s \right) \quad (5.11)$$

To limit the pressure angle ' α ' to the value less than 30° , ϕ_s which is angle of rotation of the cam during sliding must be atleast equal to 70° . In the present example, the angle ' ϕ_s ' is taken to be 70° for which α comes to be equal to 27° .

5.2 CAM FOLLOWER SYNTHESIS WHEN THERE IS PURE ROLLING

In this part, the equation representing the follower motion is given as

$$b = \frac{L_r}{\phi_r^n} \phi^n \quad (5.12)$$

The exponent ' n ' in the equation 5.12 must always be taken greater than unity. This is in conformity with the limitations discussed in Chapter 2.

Now, differentiating (5.12) once with respect to ϕ , it gives

$$b' = n \frac{L_r}{\phi_r^n} \phi^{n-1} \quad (5.13)$$

But b' at $\phi = \phi_r$ is ϕ_1 , therefore (5.13) gives

$$\phi_1 = n \cdot \frac{L_r}{\phi_r^n} \phi_r^{n-1}$$

$$\text{or } \phi_1 = n \cdot \frac{L_r}{\phi_r^n}, \text{ which gives}$$

$$n = \frac{\phi_1 \phi_r}{L_r} \quad (5.13)$$

Substituting the value of ϕ_1 from (5.5) into (5.14), it gives

$$n = \frac{\pi}{2} \frac{L_s}{L_r} \frac{\phi_r}{\phi_s} \quad (5.14)$$

5.3 NUMERICAL EXAMPLE

For a total rise of 4 cm during 150° angle of cam rotation, the cam contour and the follower surface are synthesised, for different percentages contributed by rolling and sliding. The cam contours and the follower surfaces are shown in the Figs. 5.1, 5.2 and 5.3.

In each case the work loss in the guide due to friction is calculated. In case of sliding portion the interface loss between the cam and follower has also been calculated. The results are presented in the table 3.1 and plotted in the Fig. 5.4.

Referring to the Fig. 5.4, it has been noticed that for a given lift, the percentage contributions by rolling action and sliding portion fall in a zone in view of minimum work losses.

CHAPTER 6

RESULTS, DISCUSSION AND CONCLUSION

6.1 RESULTS AND DISCUSSION

The cam contour and the corresponding follower surface have been synthesised for pure rolling contact during rise period of a dwell-rise-dwell-return cam. It is found that under present circumstances (dwell-rise-dwell-return-cam), the total lift of the follower cannot be achieved by pure rolling contact. Therefore, a fraction of the lift is allowed to be contributed by rolling cum sliding contact between the cam and the follower surfaces. For sliding contact also cam contour is synthesised, taking follower face to be flat.

In cam-follower action, work-losses are incurred primarily at the cam - follower interface during the sliding contact and in the guide. It is found that if total lift of the follower is contributed by sliding contact alone, the work-losses are enormous. But partial contribution by rolling contact brings down the work losses. This proves the advantages of providing rolling contact. As the percentage contribution of lift by rolling contact is increased, it is found that the work-losses start increasing beyond a certain percentage (in the discussed numerical example it is 37.5%). This is due to the observed increase in pressure angle.

However, this result is being depicted on the basis of one numerical example. But it is expected that the work-losses will decrease, if the percentage contribution by rolling contact increased up to a practicable value, keeping other parameters within a normal limit.

6.2 CONCLUSIONS

From the present work on cam-follower synthesis with rolling contact, following conclusions can be drawn:

1. Though mathematically it is possible to synthesise cam and the corresponding follower contours for entire rise period of a dwell-rise-dwell-return cam, but for the transfer of motion a fraction of the lift (later part) has to be contributed by rolling-cum-sliding contact.
2. In view of minimum work-loss, the respective percentage contribution by rolling contact and sliding action for the total lift can be worked out.

TABLE 1.1 : Flat - Face Follower

The numerical values are as follows:

$$\alpha = 150^\circ$$

$$m = \tan \alpha = -0.5774,$$

$$a_0 = 2 \text{ cm}$$

Hence, equation 2.45 becomes

$$\rho = 2 e^{1.732 \phi}$$

S.No.	ϕ (deg.)	ρ (cm)
1	0	2.000
2	10	2.706
3	20	3.662
4	30	4.952
5	40	6.702
6	50	9.066
7	60	12.266

Cam and the corresponding follower contour are shown in the Fig. 2.5.

TABLE 1.2 : Follower - Face Parabolic

The numerical values are as follows:

$$a_o = 2 \text{ cm.}$$

$$p = 4 \text{ cm.}$$

and $m_o = 2 \text{ cm}$, which gives

$$l_o = 5.656 \text{ cm.},$$

$$k_o = 7.656 \text{ cm.},$$

and $d = 4 \text{ cm.}$

Solution of equation 2.58 gives

S.No.	ϕ (deg.)	r (cm)
1	0	2.00
2	10	2.60
3	20	3.55
4	30	5.10

The cam and the corresponding follower contour is shown in the Fig. 2.6.

TABLE 1.3 : Follower Motion : Parabolic

Numerical values are as follows

$$a_o = 2 \text{ cm},$$

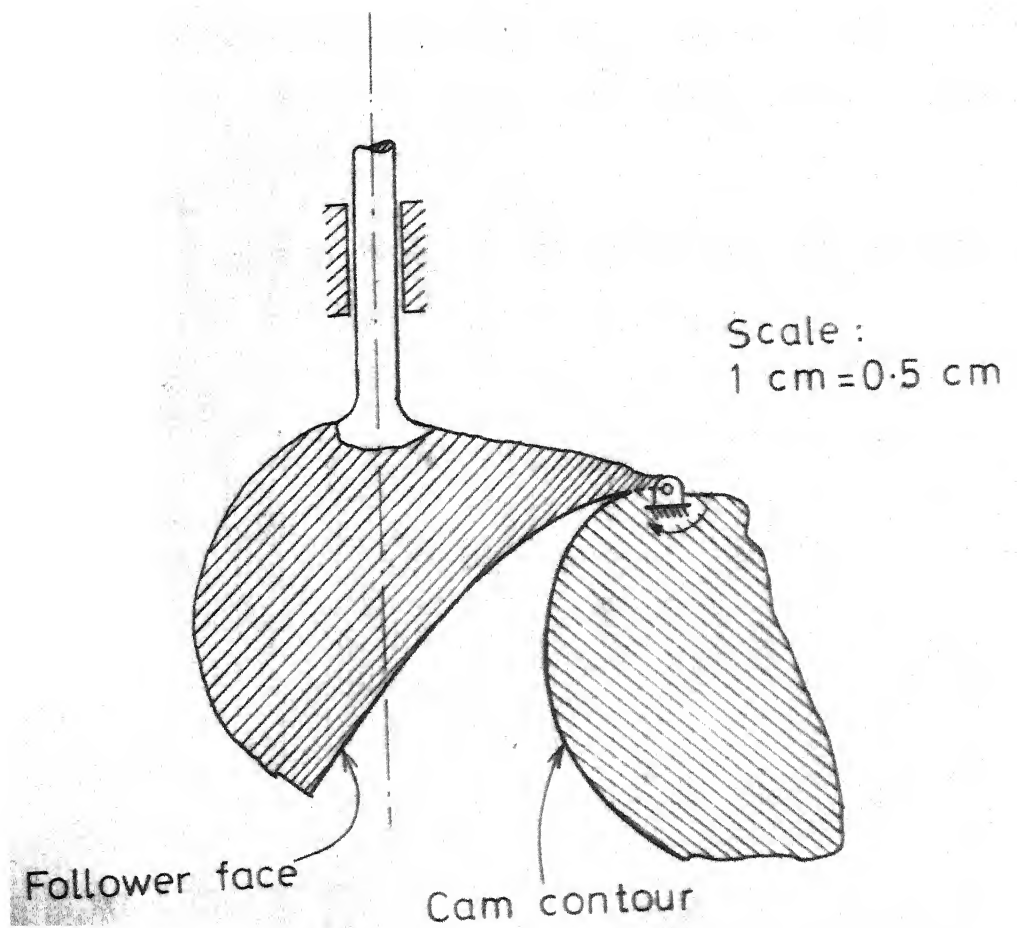
$$L_r = 2 \text{ cm}$$

$$\text{and } \phi_r = 90^\circ$$

S.No.	ϕ (deg)	b (cm)	b' (cm/rad)	X (cm)	Y (cm)
1	0	0.000	0.000	2.000	0.000
2	10	0.025	0.283	1.717	0.025
3	20	0.099	0.566	1.434	0.099
4	30	0.222	0.849	1.151	0.222
5	40	0.395	1.132	0.868	0.395
6	50	0.617	1.415	0.585	0.617
7	60	0.889	1.698	0.302	0.889
8	70	1.210	1.981	0.019	1.210
9	80	1.580	2.264	-0.264	1.580
10	90	2.000	2.546	-0.546	2.000

Cam and the corresponding contours are shown in

Fig. 2.7.



FOLLOWER MOTION : PARABOLIC

FIG. 2.7

TABLE 1.4 : Follower Motion : SHM

The numerical values are as follows:

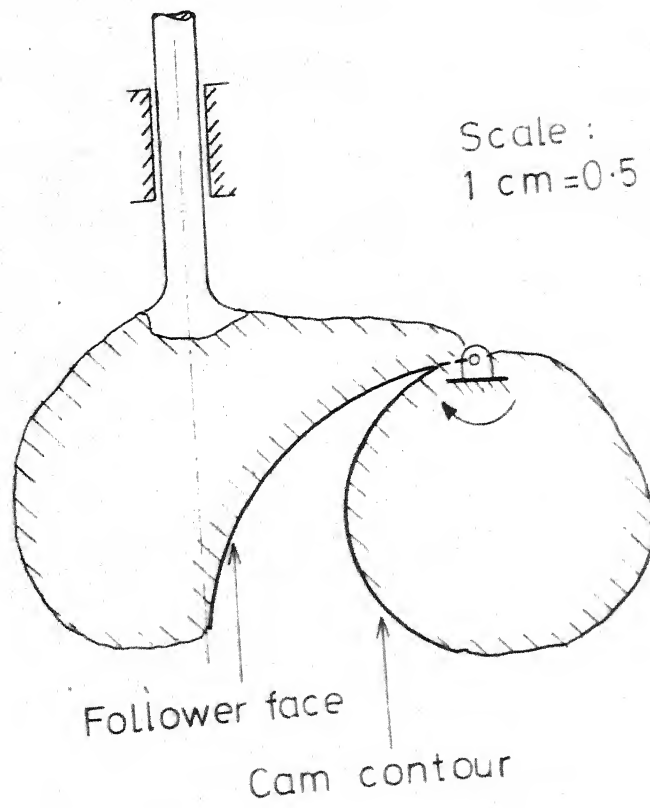
$$a_o = 2 \text{ cm},$$

$$L_r = 4 \text{ cr}$$

$$\text{and } \phi_r = 180^\circ$$

S.No.	ϕ (deg)	b (cm)	b' (cm/rad)	X (cm)	Y (cm)
1	0	0.000	0.000	2.000	0.000
2	10	0.030	0.347	1.653	0.030
3	20	0.121	0.684	1.316	0.121
4	30	0.268	1.000	1.000	0.268
5	40	0.468	1.287	0.713	0.468
6	50	0.714	1.532	0.468	0.714
7	60	1.000	1.732	0.268	1.000
8	70	1.316	1.879	0.121	1.316
9	80	1.653	1.970	0.030	1.653
10	90	2.000	2.000	0.000	2.000

Cam and the corresponding follower contour is shown in Fig. 2.8.



FOLLOWER MOTION : S.H.M

FIG. 2.8

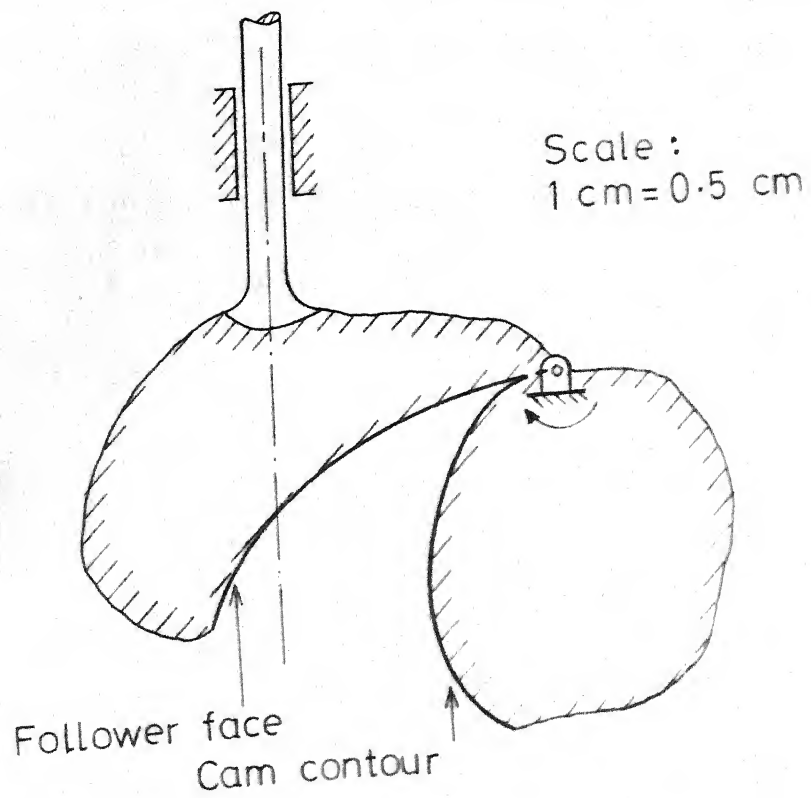
TABLE 1.5 : Follower Motion : Cycloidal

The numerical values are as follows:

$$\begin{aligned} a_o &= 2 \text{ cm,} \\ L_r &= 4 \text{ cm} \\ \text{and } \phi_r &= 180^\circ \end{aligned}$$

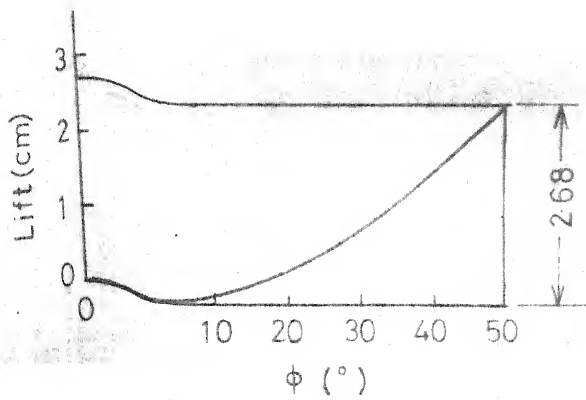
S.No.	ϕ (deg)	b (cm)	b' (cm/rad)	X (cm)	Y (cm)
1	0	0.000	0.000	2.000	0.000
2	10	0.004	0.077	1.923	0.004
3	20	0.035	0.298	1.702	0.035
4	30	0.115	0.637	1.363	0.115
5	40	0.262	1.052	0.948	0.262
6	50	0.484	1.494	0.506	0.484
7	60	0.782	1.910	0.090	0.782
8	70	1.146	2.247	-0.24748	1.146
9	80	1.560	2.470	-0.470	1.560
10	90	2.000	3.744	-0.744	2.000

Cam and the corresponding follower contour are shown in Fig. 2.9.



FOLLOWER MOTION: CYCLOIDAL

FIG 2-9



DISPLACEMENT DIAGRAM

FIG. 2-10

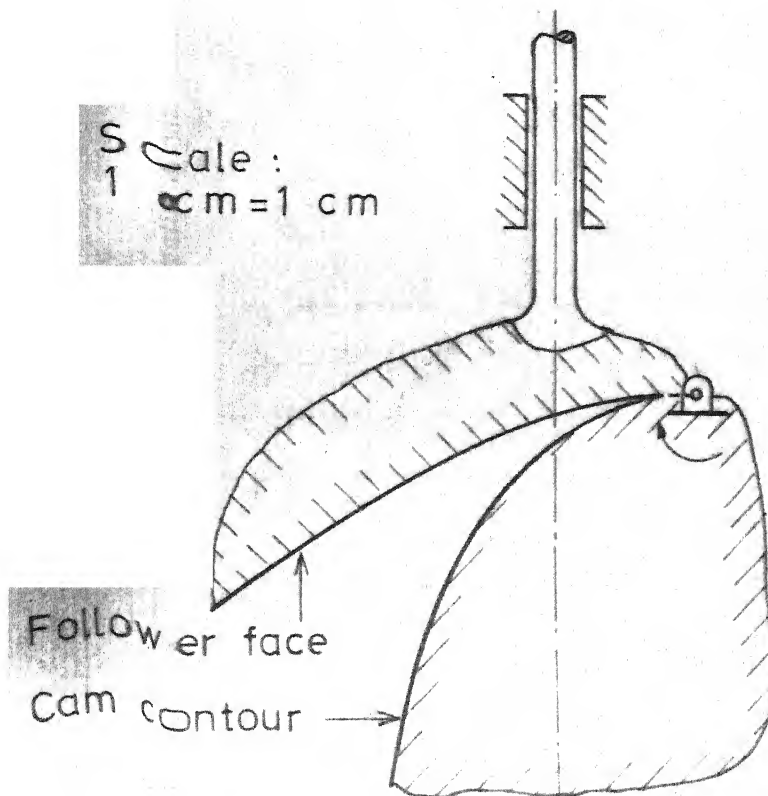


FIG. 2-11

NUMERICAL EXAMPLES : CAM-FOLLOWER SYNTHESIS

TABLES 2.1

The numerical values used are

$$\begin{aligned} L_r &= 1 \text{ cm}, & L_s &= 3 \text{ cm}, \\ \phi_r &= 80^\circ, & \phi_s &= 70^\circ \\ \text{and } a_o &= 2 \text{ cm}. \end{aligned}$$

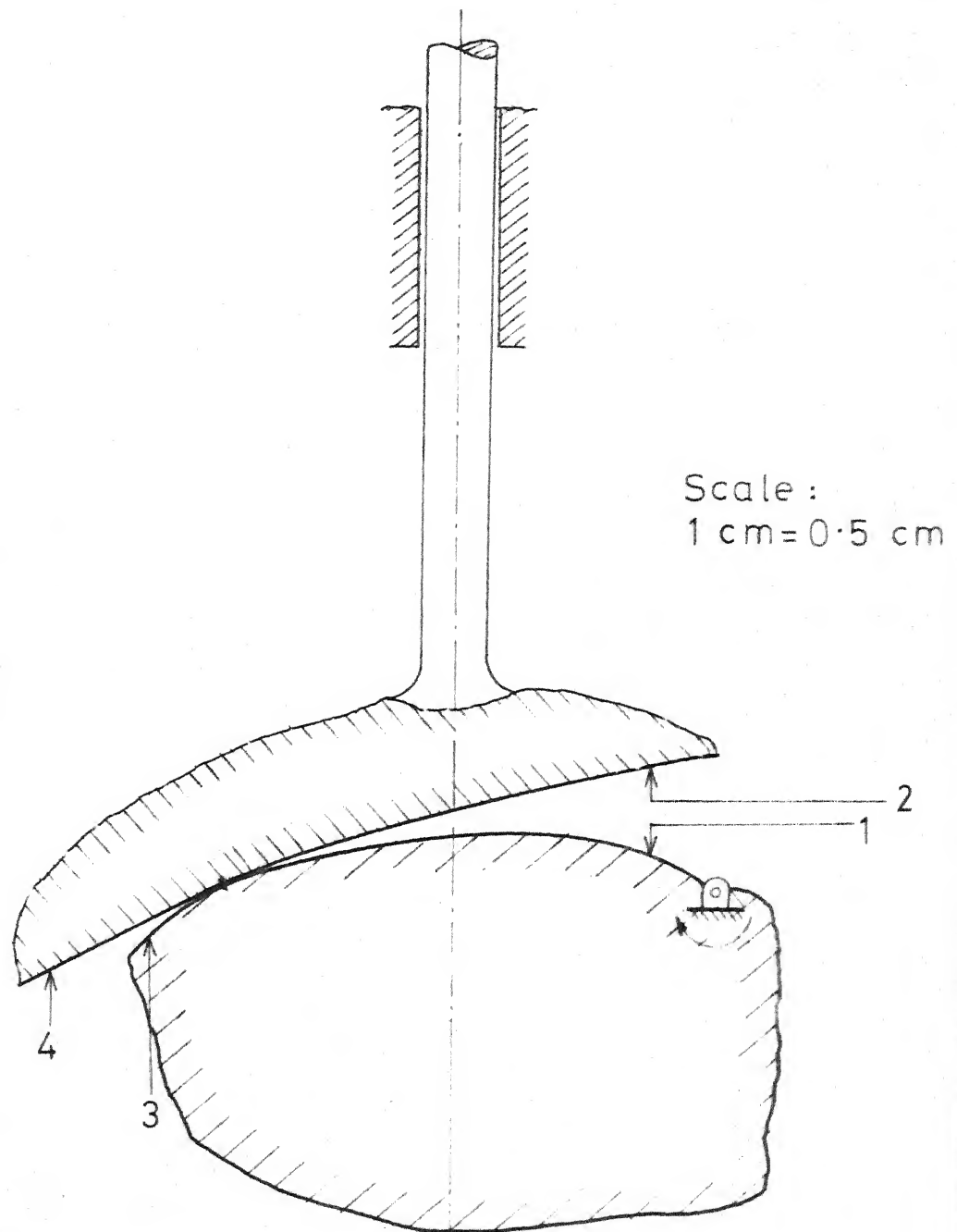
a) Synthesis for Rolling Part

S.No.	ϕ (deg)	b (cm)	b' (cm/rad)	X (cm)	Y (cm)
1	0	0.0000	0.0000	2.0000	0.0000
2	10	0.0001	0.0004	1.9996	0.0001
3	20	0.0006	0.0008	1.9992	0.0006
4	30	0.0051	0.0523	1.9477	0.0051
5	40	0.0239	0.1845	1.8155	0.0239
6	50	0.0796	0.4910	1.5090	0.0796
7	60	0.2124	1.0922	0.9078	0.2124
8	70	0.4871	2.1473	-0.1473	0.4871
9	80	1.0000	3.8569	-1.8569	1.0000

b) Synthesis for Sliding Portion

S.No.	ϕ (deg)	b (cm)	\bar{x} (cm)	\bar{y} (cm)
1	0	0.0000	-1.8571	0.0000
2	10	0.6676	-2.0877	0.1429
3	20	1.3016	-2.2406	0.2793
4	30	1.8704	-2.3297	0.3920
5	40	2.3455	-2.3729	0.4716
6	50	2.7029	-2.3882	0.5173
7	60	2.9248	-2.3912	0.5357
8	70	3.0000	-2.3912	0.5392

Cam and the follower contours are shown in Fig. 5.1.



Notations:

- 1 Cam contour for rolling contact
- 2 Follower face for rolling contact
- 3 Cam contour when sliding is present
- 4 Follower face when sliding is present

FIG. 5.1

TABLES 2.2

The numerical values used are

$$\begin{aligned} L_r &= 1.5 \text{ cm}, & L_s &= 2.5 \text{ cm}, \\ \phi_r &= 80^\circ, & \phi_s &= 70^\circ \\ \text{and } a_o &= 2 \text{ cm}. \end{aligned}$$

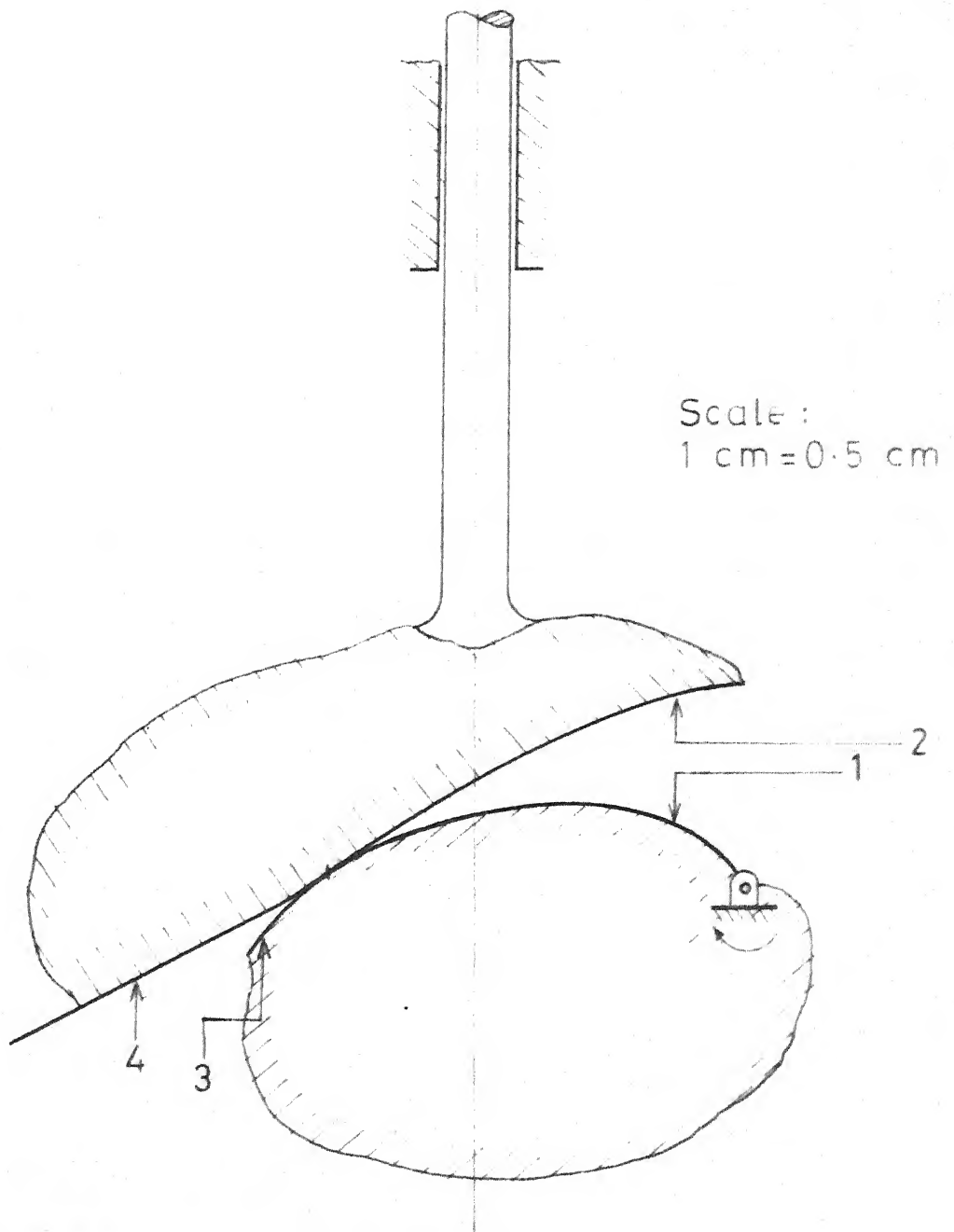
a) Synthesis for Rolling Part

S.No.	ϕ (deg)	b (cm)	b' (cm/deg)	X (cm)	Y (cm)
1	0	0.0000	0.0000	2.0000	0.0000
2	10	0.0030	0.0511	1.9489	0.0030
3	20	0.0237	0.2031	1.7969	0.0237
4	30	0.0797	0.4556	1.5444	0.0797
5	40	0.1885	0.8080	1.1920	0.1885
6	50	0.3676	1.2603	0.7397	0.3676
7	60	0.6343	1.8122	0.1878	0.6343
8	70	1.0059	2.4635	-0.4635	1.0059
9	80	1.5000	3.2142	-1.2142	1.5000

b) Synthesis for Sliding Part

S.No.	ϕ (deg)	b (cm)	\bar{x} (cm)	\bar{y} (cm)
1	0	0.0000	-1.2143	0.0000
2	10	0.5563	-1.4064	0.1191
3	20	1.0847	-1.5339	0.2328
4	30	1.5587	-1.6081	0.3267
5	40	1.9546	-1.6441	0.3930
6	50	2.2524	-1.6569	0.4311
7	60	2.4373	-1.6593	0.4464
8	70	2.5000	-1.6593	0.4493

Cam and the corresponding follower contours are shown in the Fig. 5.2



Notations:

- 1 Cam contour for rolling contact
- 2 Follower face for rolling contact
- 3 Cam contour when sliding is present
- 4 Follower face when sliding is present

FIG . 5-2

TABLES 2.3

The numerical values used are

$$\begin{aligned} L_r &= 2 \text{ cm}, & L_s &= 2 \text{ cm}, \\ \phi_r &= 80^\circ, & \phi_s &= 70^\circ \\ \text{and } a_o &= 2 \text{ cm}. \end{aligned}$$

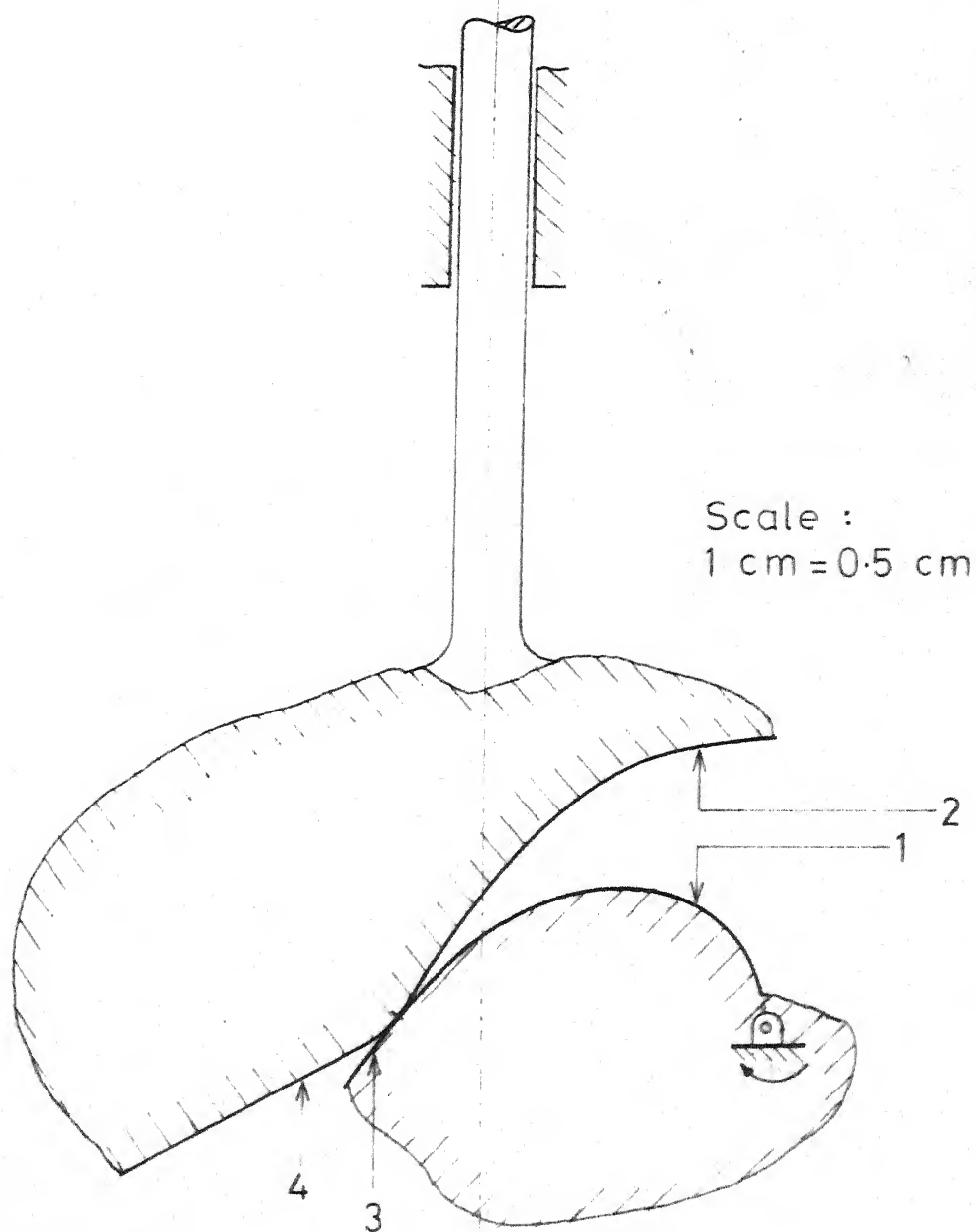
a) Synthesis for Rolling Part

S.No.	ϕ (deg)	b (cm)	b' (cm/deg)	X (cm)	Y (cm)
1	0	0.0000	0.0000	2.0000	0.0000
2	10	0.0478	0.4921	1.5079	0.0478
3	20	0.1660	0.8539	1.1461	0.1660
4	30	0.3438	1.1786	0.08212	0.3438
5	40	0.5763	1.4818	0.5182	0.5763
6	50	0.8602	1.7695	0.2305	0.8602
7	60	1.1933	2.0456	-0.0456	1.1933
8	70	1.5737	2.3124	-0.3124	1.5737
9	80	2.0000	2.5714	-0.5714	2.0000

b) Synthesis for Sliding Part

S.No.	ϕ (deg)	b (cm)	\bar{x} (cm)	\bar{y} (cm)
1	0	0.0000	-0.5714	0.0000
2	10	0.4459	-0.7213	0.0953
3	20	0.8676	-0.8271	0.1862
4	30	1.2470	-0.8865	0.2614
5	40	1.5636	-0.9153	0.3144
6	50	1.8019	-0.9255	0.3448
7	60	1.9499	-0.9274	0.3571
8	70	2.0000	-0.9275	0.3595

Cam and the corresponding follower contours are shown in Fig. 5.3



Notations:

- 1 Cam contour for rolling contact
- 2 Follower face for rolling contact
- 3 Cam contour when sliding is present
- 4 Follower face when sliding is present

FIG.5.3

TABLE 3.1 : ESTIMATION OF WORK - LOSSES

The numerical values are as follows:

$$\begin{aligned}
 L &= 4 \text{ cm}, & a_o &= 2 \text{ cm}, & \phi_o &= 150^\circ \\
 P_o &= 10 \text{ kgf}, & k &= 40 \text{ kgf/cm}, & \mu &= 0.25, \\
 \mu_i &= 0.3 & h &= 4 \text{ cm} & l &= 8 \text{ cm}.
 \end{aligned}$$

S.No.	L_r cm	L_s cm	ϕ_r°	ϕ_s°	W_{gr} kgf-cm	W_{gs} kgf-cm	W_i kgf-cm	W_s kgf-cm
1	1.0	3.0	80	70	8.6368	3.8702	16.1421	20.0123
2	1.5	2.5	80	70	30.7696	2.1136	10.9984	13.1119
3	2.0	2.0	80	70	387.1607	1.0198	7.0365	8.0563
4	0.0	4.0	0	150	0	2.8069	255.3097	258.3097

Notations used are as follows:

W_{gr} = Work-loss in the guide during rolling contact

W_{gs} = Work-loss in the guide during sliding part

W_i = Work loss at the cam follower interface during sliding

W_s = Total work loss during sliding portion